Stereo Disparity Estimation in Moment Space

Angeline Pang
Faculty of Information Technology, Multimedia University,
63100 Cyberjaya, Malaysia.
angeline.pang@mmu.edu.my

R. Mukundan
Department of Computer Science, University of Canterbury,
Christchurch, New Zealand.
mukund@cosc.canterbury.ac.nz

Ng Liang Shing
Faculty of Engineering, Multimedia University,
63100 Cyberjaya, Malaysia.
lsng@mmu.edu.my

Abstract

This paper explores various ways by which pixel disparities along epipolar rows of a stereo image pair could be obtained using Chebyshev moments of the corresponding 1D images. Chebyshev moments allow us to represent the image features based on orthogonal basis functions, and hence to reconstruct the intensity distribution from a set of moments fairly accurately. Disparity estimates can be obtained either analytically from the inverse moment transform, or by comparing the reconstructed intensity values. The paper also presents some preliminary results obtained using both synthetic and real images.

Keywords: Stereo Image Processing, Disparity Estimation, Discrete Orthogonal Moments, Chebyshev Polynomials.

1 Introduction

Moment functions such as geometric and Zernike moments, are used in several computer vision applications for representing global and invariant shape characteristics of image features. Orthogonal moments (Zernike, Pseudo-Zernike etc.) have been proved to be less sensitive to image noise when compared to geometric moments, and possess far better feature representation capabilities [1-3]. For example, the information redundancy measure is minimum in an orthogonal moment set. Recently, discrete orthogonal moments based on Chebyshev (Tchebichef) polynomials were introduced [4-6], which can also be used as image descriptors with minimum information redundancy. Chebyshev moments are orthogonal in the image coordinate space itself, and thus provide more accurate reconstructions of the original image; while the computation of Zernike moments involves discrete approximations and coordinate transformations which severely affect their orthogonality properties in the image space.

Since the image intensity distribution can be very accurately represented by Chebyshev moments, the matching of intensity values in applications such as stereopsis, can also be attempted in the moment space. Along epipolar stereo image rows, the correspondence between two intensity values $I(x)$ and $I(x+d_x)$ is represented by the disparity d_x at pixel x. Using discrete orthogonal moments, we can have an accurate polynomial approximation of the intensity distribution $I(x)$ in terms of the kernel functions (Chebyshev polynomials of degree p) $t_p(x)$. Thus, in the moment space, the correspondence between two pixels in a stereo pair can be expressed as a relation involving terms $t_p(x+d_x)$ of different orders p, where the coefficients are moments computed from the two images. The analytical characteristics of the kernel functions are well known. We can therefore try to implement some novel approaches to disparity estimation, such as

- obtaining the values of d_x by comparing moment terms rather than by comparing intensity values.
- deriving analytical solutions for d_x from moment equations.

The above considerations were the main motivation behind the work reported in this paper. The next section outlines the framework of Chebychev moments and their inverse moment transforms. A few analytical derivations of the disparity values from moment equations are given in Section 3. Experimental results and a discussion on the advantages and limitations of moment based stereopsis are given in Section 4.

2 Chebyshev Moments

Given an NxN image, discrete orthogonal polynomials $\{t_n(x)\}$ over the image coordinate space satisfy the condition

$$\sum_{x=0}^{N-1} t_m(x) t_n(x) = \rho(n,N)\delta_{mn}\ , \qquad m, n = 0,1,\ldots N-1. \quad (1)$$

where $\rho(n,N)$ is the squared norm of the polynomial set t_n. The classical discrete Chebyshev polynomials [7,8] satisfy the above property of orthogonality (1). For image analysis applications, we can appropriately scale the above polynomials to avoid numerical instabilities in the computation of large-order moments. Some of the possible sets of scale factors are discussed in [4]. Alternatively, we can also make the polynomial functions orthonormal with $\rho(n,N)=1$, (which also amounts to introducing a scale factor) and minimize the propagation of errors using the process of renormalization [9].

The Chebyshev moments of order $p+q$ are defined as [4-6]

$$T_{pq} = \frac{1}{\rho(p,N)\rho(q,N)} \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} t_p(x) t_q(y) f(x,y)\ ,$$

$$p, q = 0, 1, \ldots N-1. \quad (2)$$

and, one set of scaled Chebyshev polynomials $t_p(x)$ can be computed using the following recurrence relation:

$$t_0(x) = 1. \quad (3)$$

$$t_1(x) = \frac{2x+1-N}{N} \quad (4)$$

$$t_p(x) = \frac{(2p-1)t_1(x)t_{p-1}(x) - (p-1)\left\{1-\frac{(p-1)^2}{N^2}\right\}t_{p-2}(x)}{p}\ ,$$

$$p>1 \quad (5)$$

The image intensity function $f(x, y)$ has a polynomial representation given by

$$f(x, y) = \sum_{p=0}^{N-1}\sum_{q=0}^{N-1} T_{pq} t_p(x) t_q(y) \quad (6)$$

where the coefficients T_{pq} are the Chebyshev moments defined in (2).

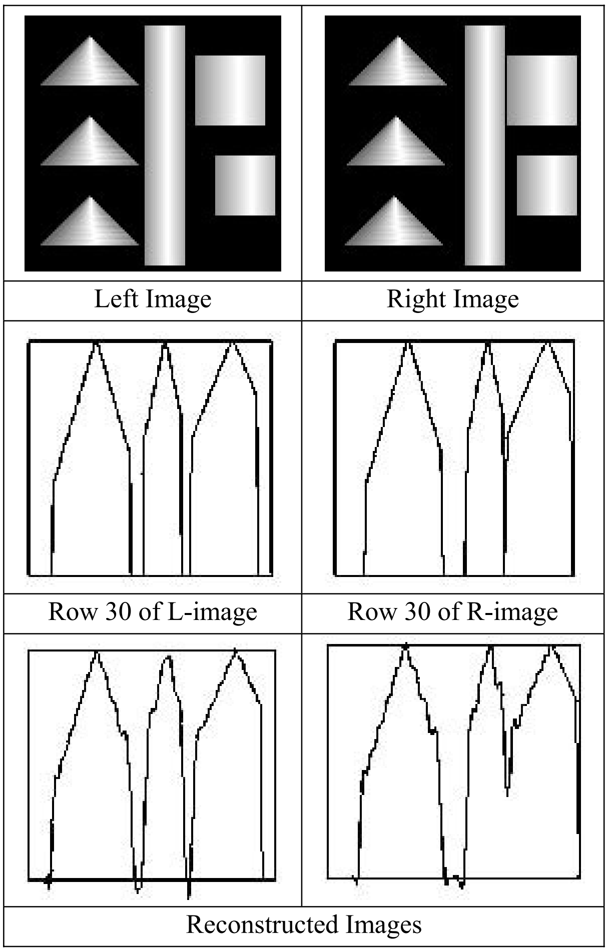

Figure 1: Original and reconstructed intensity distributions on an image row of a stereo pair.

We can easily write the one-dimensional versions of equations (2), (6) as

$$T_p = \frac{1}{\rho(p,N)} \sum_{x=0}^{N-1} t_p(x) f(x), \quad p=0,1,\ldots N-1. \quad (7)$$

and, $$f(x) = \sum_{p=0}^{N-1} T_p t_p(x). \quad (8)$$

Figure 1 gives an example of a synthetic stereo image pair, and the original and reconstructed intensity distributions on a particular row.

3 Disparity Values and Moment Equations

In this section, we attempt to derive analytical relationships between disparity values and the intensity moments using the equations given above. If we denote the one-dimensional intensity distributions along epipolar lines of the left and the right image by $f_l(x)$ and $f_r(x)$ respectively, then a disparity value d_x at the pixel x establishes a correspondence in the intensity domain given by

$$f_l(x) = f_r(x+d_x) \quad (9)$$

If we use the polynomial representation of the intensity function as given in (8), we get

$$\sum_{p=0}^{M} \left(T_p^{(r)} t_p(x+d_x) - T_p^{(l)} t_p(x) \right) \approx 0.0 \quad (10)$$

where $T_p^{(l)}$, $T_p^{(r)}$ denote the p^th^ order Chebychev moments of intensity functions (7) along the left and the right epipolar image rows, and M is a sufficiently large value (typically $N/2$ or greater) denoting the maximum order of moments.

Several analytical representations for the disparity function d_x can be derived from (10). For example, we can write

$$d_x = \frac{\sum_{p=0}^{M} \left(T_p^{(l)} - T_p^{(r)} \right) t_p(x)}{\sum_{p=0}^{M} T_p^{(r)} t'_p(x)} \quad (11)$$

where $t'_p(x)$ denotes the derivative of $t_p(x)$. A p^{th} order polynomial will have p zeros, and hence for large values of p, the derivative of the function $t_p(x)$ will have many points where the value is either zero or is very large. Thus (11) will not, in many cases, provide any meaningful results. An alternative approach is to compute the coarse disparity estimates using low-order moments (using(3),(4)) as

$$d_x = \frac{N}{2T_1^{(r)}} \left[\left(T_0^{(l)} - T_0^{(r)} \right) + \left(T_1^{(l)} - T_1^{(r)} \right) \frac{2x+1-N}{N} \right] \quad (12)$$

and then develop an iterative update scheme for d_x using higher order moments, applying the constraint (10). The plot of d_x for a fixed value of x is given in figure 2.

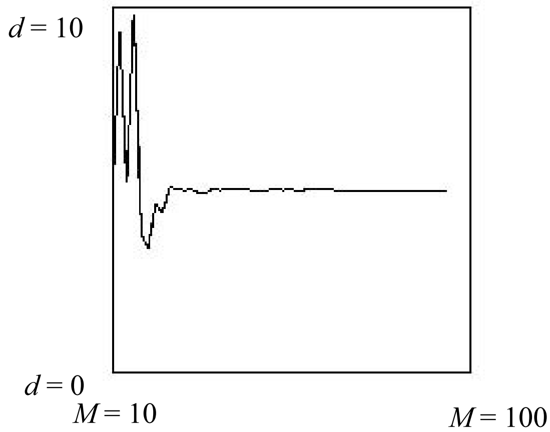

Figure 2: Variation of disparity with respect to coarse-to-fine reconstruction using moments.

4 Experimental Analysis

Instead of using the analytical approaches detailed in the previous section, we could also attempt to estimate disparity values in the moment space, by minimizing the difference between reconstructed intensity values: In other words, for each x, we seek a value d_x which minimizes

$$\lambda_x = \sum_{p=0}^{M} \left(T_p^{(r)} t_p(x+d_x) - T_p^{(l)} t_p(x) \right)^2 \quad (13)$$

Assuming that the disparity function is continuous in a neighborhood $[x-k, x+k]$ of x for small values of k, we can redefine the cost function as:

$$\lambda_x = \sum_{i=-k}^{k} \left(\sum_{p=0}^{M} \left(T_p^{(r)} t_p(x+d_x+i) - T_p^{(l)} t_p(x+i) \right)^2 \right) \quad (14)$$

The disparity values obtained for the 30^th^ row of the image pair in figure 1., and for the whole image, are shown below.

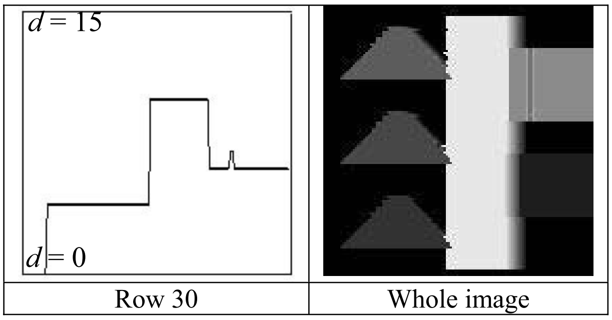

Figure 3: Disparity map for the stereo pair in figure1.

The primary advantage of moment based stereo disparity estimation is that the disparity values can be represented as offsets in the arguments of kernel functions, whose characteristics are well known (unlike the intensity distribution). This allows us to choose cost functions that depend on the maximum order of moments used. The reconstructed polynomial approximation of the intensity values, can exhibit Gibb's phenomenon (as in the case of inverse Fourier transform) near regions of large intensity gradients. Such effects will have to be eliminated for improving the accuracy of the estimates. Figure 4 shows the reconstruction of row 42 of the left-image in figure 1, and the effect of the Gibb's phenomenon on disparity estimates.

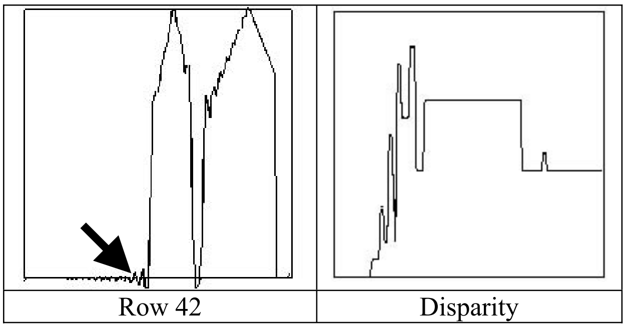

Figure 4: Effect of Gibb's phenomenon on disparity values.

The polynomial approximation of the intensity function can reduce the effects of image noise. However, since an entire row is represented by a single function, background intensities will not be separable from those of the foreground objects. This can also be observed in figure 1. Such a blending of intensities might lead to erroneous estimates. Figure 6 gives the initial experimental results with real images in figure 5. Further improvements in the disparity estimates can be obtained using a dynamic programming approach and rectangular windows.

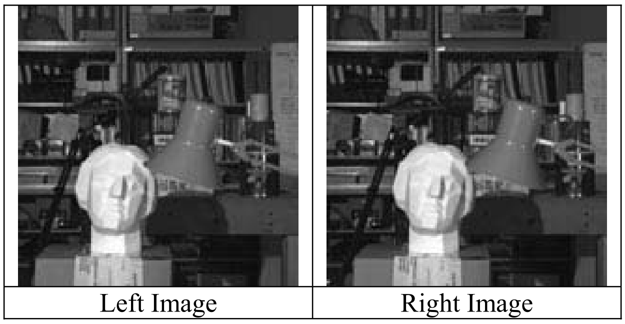

Figure 5. Real images used in experimental analysis.

Figure 6. Disparity map obtained using images in figure 5.

Some possible extensions of the work reported in this paper are listed below.

- A progressive update of disparity values using (12) with local smoothness constraints.
- Combining the minimization problem (13) with a dynamic programming approach [10] for refining the disparity map.
- Application of a coarse-to-fine reconstruction strategy together with coarse-to-fine dynamic programming [11] to minimize the computation time.

5 Conclusions

The paper has presented a method based on discrete Chebyshev moments using which the disparity values in a stereo image pair could be represented in the moment space. Such a representation allows pixel disparities to be equivalently denoted by offsets in the argument of continuous polynomial functions. Consequently, several analytical as well as numerical characterizations of the disparity values could be possible, given a large set of equations involving moments (of various orders) of the left and right images. The possibility of using this approach in stereopsis has been explored, and some experimental results were presented.

6 References

[1] M.R. Teague: Image analysis via the general theory of moments, *J. of Opt. Soc. of America* **70** (1980), 920–930.

[2] S.O. Belkasim, "Pattern recognition with moment invariants – A comparative study and new results," *Pattern Recognition*, vol. 24, pp. 1117-1138, 1991..

[3] R. Mukundan, K.R. Ramakrishnan: *Moment Functions in Image Analysis-Theory and Applications*, World Scientific, Singapore (1998).

[4] R. Mukundan, S. H. Ong, P. A. Lee, "Image analysis by Tchebichef moments", *IEEE Trans. Image Processing*, vol.10 (9), pp.1357-1364 (2001).

[5] R. Mukundan, S. H. Ong, P. A. Lee, "Discrete orthogonal moment features using Chebyshev polynomials," *Proc. Image and Vision Computing New Zealand* IVCNZ-2000, pp. 20-25, 2000.

[6] R. Mukundan, S. H. Ong, P. A. Lee, "Discrete vs. Continuous Orthogonal Moments for Image Analysis," *Proc. Intl. Conf. Imaging Science Systems and Technology* CISST-2001, vol. 1, pp. 23–29, 2001.

[7] Erdelyi, A. et al , *Higher Transcendental Functions*, Vol. 2, McGraw Hill, (New York: 1953).

[8] Nikiforov A.V, Suslov S.K, Uvarov V.B, *Classical Orthogonal Polynomials of a Discrete Variable*, Springer-Verlag (1991).

[9] R.Mukundan, "Improving Image Reconstruction Accuracy Using Discrete Orthonormal Moments", *Proc. Intl. Conf. Imaging Science Systems and Technology* CISST-2003, vol. 1, pp. 287–291, 2003.

[10] J.Cox, *et.al*. "A maximum likelihood stereo algorithm", Computer Vision and Image Understanding, Vol. 63(3), pp. 542-567.

[11] C.Raphael, "Coarse-to-fine dynamic programming", IEEE Trans. On Pattern Analysis and Machine Intell., vol. 23 (12), pp. 1379-1390 (2001).