Extraction of Shape of an Object and Construction of closed Curves using Image Information

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Abstract

A mesh generating system has been developed in order to prepare large amounts of input data which are needed for easy implementation of a finite element analysis. This system consists of a Pre-Mesh Generator, an Automatic Mesh Generator and a Mesh Modifier. Pre-Mesh Generator produces the shape and sub-block information as input data of Automatic Mesh Generator by carrying out various image processing with respect to the image information of the drawing input using a scanner. Automatic Mesh Generator generates mesh of triangular elements in the arbitrarily shaped and multiple connected planar domain using minimum necessary information. This generator has 3 methods of mesh generation for each sub-block. Any of them can be selected automatically according to the external form of sub-block or the state of domain. Mesh Modifier projects and modifies the pattern of generated mesh by Automatic Mesh Generator as required.

Keywords: finite element analysis, mesh generator, construction of closed curve, image information

1 Introduction

A mesh generating system has been developed as an easy implementation to prepare large amounts of input data, which are needed for finite element analysis. This system consists of a Pre-Mesh Generator, an Automatic Mesh Generator and a Mesh Modifier. Automatic Mesh Generator generates mesh of triangular elements in the arbitrarily shaped and multiple connected planar domain by using minimum necessary information. Then, with this method it is possible to effectively adapt to transformation of a shape of domain through the process of shape optimization[1]. Mesh Modifier projects and modifies the pattern of generated mesh by Automatic Mesh Generator as required. The usefulness of this system has already been confirmed by actually using it in finite element analysis.

However, the shape information of object, as input data, must be prepared, even if this system is utilized. One of the advantages of the finite element method is that it can also be applied to the object of complicated arbitrary shapes[2],[3]. But a considerable time and effort is needed in order to prepare such complicated shape information.

Therefore, a Pre-Mesh Generator with such function has been developed. Image information of the drawing input by a scanner is converted into line drawing with a line-width of 1 pixel without losing the continuity[4]. Then, the line is segmented by structurally analyzing the line-drawing about its connection relation, which is the bending and branching condition of the line. Sequence of points, which can accurately reproduce the shape of object and several divided sub-blocks of the object which are defined by closed curves, are decided automatically and used as input data for the above-mentioned Automatic Mesh Generator. By adding it to this system, user's task in carrying out the finite element analysis is simplified sufficiently.

2 Outline of Pre-Mesh Generator

The outline of the process of Pre-Mesh Generator is as follows:

1. Shape and sub-block information of the analysis object are read from the drawing using a scanner.
2. The grayscale picture is converted into binary image.
3. The noise is removed by labeling treatment.
4. By thinning process, binary image is made into the linear drawing with the line width of 1 pixel without losing continuity.
5. By carrying out borderline tracking process, the twig component is removed and shape line of analysis object and boundary line between sub-blocks are extracted.
6. The line is segmented at the point of the branching (called block point).
7. The line segment is approximated by the polygonal line (Method Ⅰ and Ⅱ).

8 Shape of the sub-block is constructed by the line segments.

The algorithm of main process in Pre-Mesh Generator is shown as follows.

2.1 Thinning process

The figure component is narrowed to the line drawing with a line width of 1 pixel without losing its continuity by removing the pixels in order from the sides of the component so that the line may pass through the center of the original component. In order to judge whether it is possible to remove a pixel valued 1, which is called attention point, the connectivity number(N_C)[5] of this point

$$N_C = \sum_{k \in S} (\overline{X}_k - \overline{X}_k \overline{X}_{k+1} \overline{X}_{k+2}) \quad (1)$$

Where,

$$S = \{1,3,5,7\}, \quad \overline{X}_k = 1 - X_k, \quad X_9 = X_1$$

is obtained by formula (1). The value of N_C is 0 ~ 4. As shown in figure.1, pixel values X_1 ~ X_8 are that of the 8 pixels in the neighborhood of attention point. The number of suffix i in X_i is given anticlockwise beginning from the right hand side of the attention point X_0. Pattern examples of the pixel values of the 8-neighborhood of the attention point are shown with various values of N_C in figure.2..

According to figure 2, in the case of N_C =1 only, even if the attention point is removed,the figure component does not get separated or combined. And also it does not cause that a hole is created or the hole is disappeared. Accordingly, the attention point X_0 is deleted. Connectivity number(N_C) of the pixels on the boundary of the figure component is calculated according to formula (1). Based on the result of the calculation, the pixels are deleted one by one until it can no longer be deleted. By this process, the figure component is converted into line-drawing with a line-width of 1 pixel while retaining its continuity. For example, the figure component of bold character A is converted into "A" with a line-width of 1 pixel as shown in figure 3.

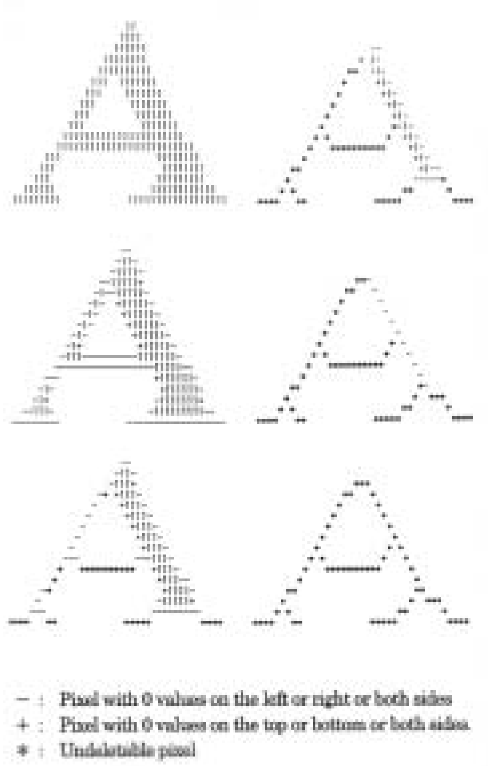


Figure 3: Example of thinning

2.2 Borderline tracking process

In borderline tracking process of the 8 connections, all closed curves made by the boundary of the figure component are extracted, while information of the image of the origin is perfectly retained. The contour is not only extracted but also the pixel on the borderline is tracked while the coordinates value of the pixel, which can redefine the contour, is examined. The borderline tracking process is carried out for the domain divided into 3 sub-blocks as shown in figure 4.

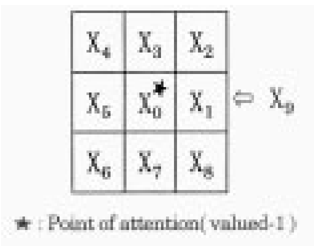


Figure 1: 8 neighborhood pixels of the point of attention

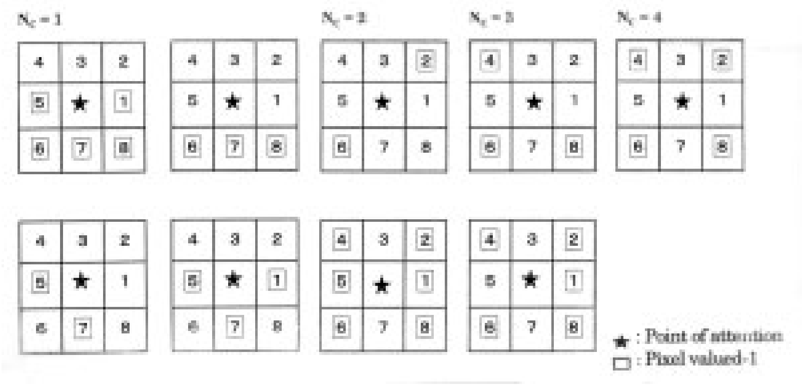


Figure: 2 Example of number of connected components related to the point of attention

With X_0 to be the starting point of the tracking process, it was possible to pick out one closed curve ACDA and two line segments ABC and BD. Next, these line segments are further segmented into 6 units between block points, and the shape of 3 sub-blocks is constructed by connecting these segments, such as ABCA, ADBA and BDCB.

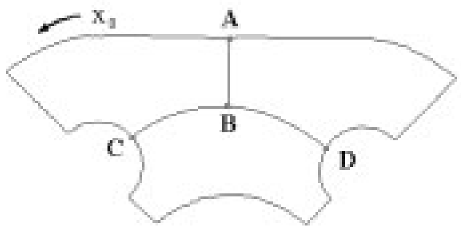


Figure 4: Domain divided into 3 sub-blocks

2.3 Polygonal approximation process

As shown in figure 5, length S along the shape of the line segment with P_0 as the starting point is defined. Here, the unit is taken as the dimensionless quantity of the length of 1 edge of 1 pixel. Line segment is composed of a large number of connecting pixels, so some representative pixels (points) are chosen from these connecting pixels and approximated. There are two methods for the purpose of the selection of representative pixels and approximation.

One of the methods is a method used to choose a representation point on a smooth curve so that the distance between representation points may become as equal as possible. The other is the method of choosing a few number of representation points on the curve complicated or with discontinuous parts, which are described as follows

(a) Method Ⅰ

First of all, the representative points are decided, so that the length between the points is approximately equal to designated length S_c and the number of representative points P_i(i = 1,2,⋯,m-1) is assumed as (m-1).

Representative point P_{i-1} and P_i(i = 1,2,⋯,m) is connected by a straight line and the point Q_i in the farthest distance from the approximate segment is selected from pixels in the segment of original curve with the ends P_{i-1} and P_i and the distance is assumed as d_i (i = 1,2,⋯,m), as shown in figure .5.
Representative points are decided as points of polygonal approximation of line segment if the maximum value d_{max} of d_i (i = 1,2,⋯,m) is within the allowable E_{max}. The number of the representative point is increased as (m+1)×0.2 at a time, when d_{max} exceeds the allowable E_{max}. However, in case the value of m is smaller than 4, the number of the representative point increased is 1. The above mentioned procedure is repeated until the condition is satisfactory. Figure 6 shows the result in the case with 3 as the initial value of m and 2.5 as E_{max}. It is possible to confirm from the result that the accuracy of the polygonal approximation of line segment can be judged by d_{max}.

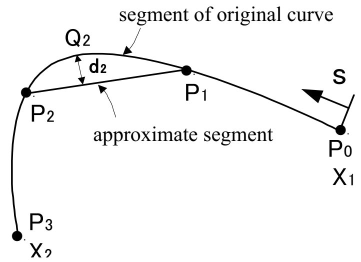


Figure 5: Polygonal approximation of line segment (m=3)

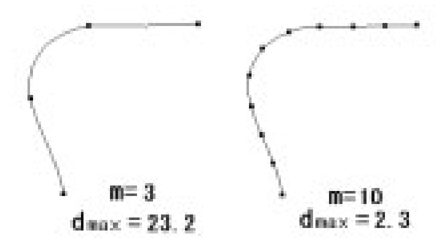


Figure 6: Points selected in order to define the configurations by Method I and d_{max} (Started as m=3, concluded as m=10)

(b) Method Ⅱ

Similarly, as mentioned in Method I , the values of d_i (i = 1,2,⋯,m) are calculated and the representative point is selected and added from the segment of original curve with P_{i-1} and P_i , in which the value of d_i exceeds the allowable E_{max}, by Method Ⅱ described as follows.

1) The point Q_i (= Q_{i1}) corresponding to d_i on the segment of original curve, where d_i exceeds the allowable E_{max} is selected and added to the representatives points and this is called point 1, as shown in figure 7(a). Dark marks● show the newly added points. Figures on the mark show the sequence in which the numbers are selected as representative points. Next, the point 1 and P_{i-1} is connected by a straight line and d_{i2} is obtained for this line segment. If the value of d_{i2} exceeds E_{max}, this point Q_{i2} for d_{i2} is added as the second additional point. Point 2 and P_{i-1}, 3 and P_{i-1}, ⋯, j and P_{i-1} are connected and d_{i3}, d_{i4}, ⋯, $d_{i\,j+1}$ are obtained for each segment, until $d_{i\,j}$ is smaller than E_{max}. Q_{ij} corresponding to d_{ij} is added in sequence to the representative points.

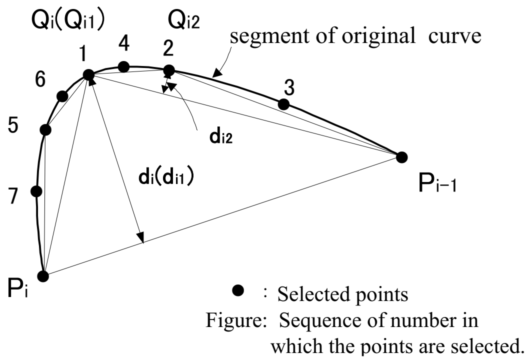


Figure 7(a): Polygonal approximation of the line segment by Method Ⅱ

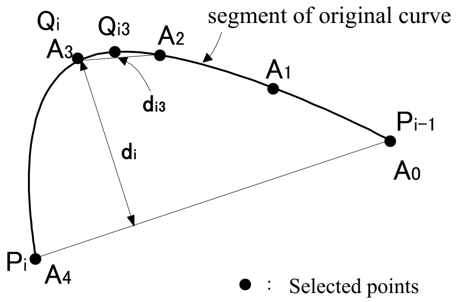


Figure 7(b): Additional points changed to serial number from P_{i-1}

2) As shown in figure 7(b), new serial number is given to the representative points added beginning from the nearest point of P_{i-1} as $A_1, A_2, \cdots, A_j$. The suffix of A shows the order from P_{i-1}.

3) d_{ij} ($j = 1,2,\cdots,j+1$) for each segment of polygonal line $P_{i-1}(= A_0)\ A_1A_2\cdots\ A_jP_i(= A_{j+1})$ is obtained, as shown in figure 7(b). In case the value of d_{ij} exceeds E_{max}, return to procedure 1) and the treatment for the line segment $P_{i-1}\ P_i$ is similarly carried out in line segment $A_{j-1}\ A_j$. This process is repeated for each segment of the polygonal line until the value of d_{ij} is less than E_{max}.

4) In the polygonal line $P_{i-1}A_1A_2,\cdots, A_j\ P_i$, in case the distance (S_d) between point A_j and P_i is smaller than the allowable distance (ε), point P_i is removed and point A_j is defined again as representative point P_i. There won't be any problem, even if point P_i is removed, because this point(pixel) has been chosen expediently as initial point on the segment of original curve as a representative point. However, point A_j is a point necessary for satisfying the condition of $d_{max} < E_{max}$. Therefore, it is not possible to delete A_j. The decision of the allowable distance between 2 points is made by $(S_d/S_c) \geqq \varepsilon$. However, this treatment is not carried out, when point P_i is the block point. The result in the case with m=3 or m=5, and E_{max}=2.5 is as shown in figure 8. It is confirmed from this result that a large number of points are designated,

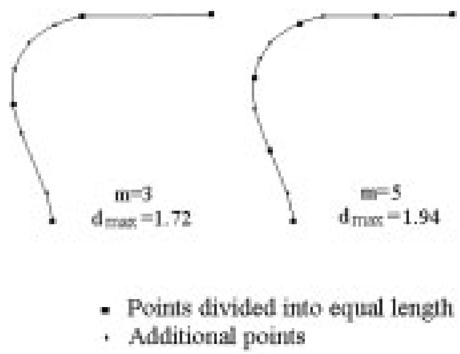


Figure 8: Points selected in order to define the configuration by Method Ⅱ and d_{max}

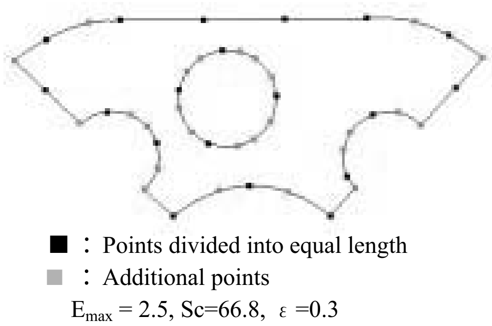


Figure 9: Selected points for defining the configurations of domain by Method Ⅱ.

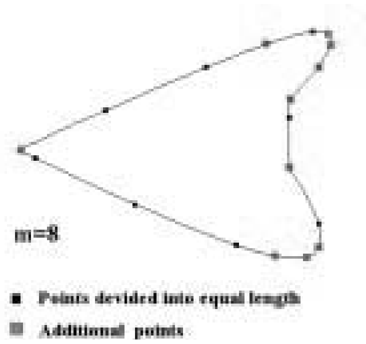
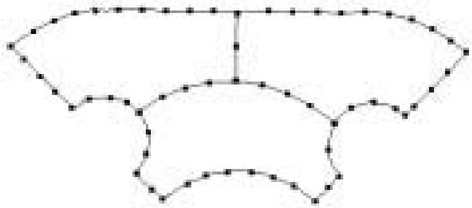


Figure 10: Example of defining the configuration of domain with a sharply pointed part by method Ⅱ

in the position where the curve changes significantly, and the shape of the line segment can be accurately expressed by these points.

The example in polygonal approximation of the shape of the multiple connected domains using Method Ⅱ is shown in figure 9. The polygonal approximation is carried out in each closed curve, because the block point does not exist for 2 closed curves of outside and inside in this example. Many discontinuous parts are contained for the closed curve of the outside, as can be seen from figure 9. When Method Ⅰ is applied in such a shape, in order to satisfy the condition $d_{max} < E_{max}$, large number of representative points (196) must be chosen. In case the same task is carried out by Method II, it is proven from figure 9 that the pixel in the discontinuous part is certainly extracted as a representative point, and the shape is accurately approximated by very little number of representative points (31) compared to Method I . And, it is confirmed that there is no great difference in the result from Method I and Method II , where the smooth curve like the inside closed curve is approximated by polygonal line. The results of applying Method II to the examples of the domain with a sharply pointed part and divided into 3 sub-blocks, with 4 block points, are shown in figure 10 and 11.



E_{max} = 2.5, Sc=22, ε =0.5

Figure 11: Selected points for defining the configurations of domain by Method II

2.4 Mesh generation process

The next step is to generate interior points and form triangular elements within each sub-block. The method of Irregular Mesh Generation (IMG)[1] proposed previously by the author, using random number, is applied for the sub-block whose form can be a multiple connected concave polygon. It has a function of controlling triangular element density in any part of the sub-block by specifying several density factors(r), as shown in figure 12. The values of density factor r, which control the size of the mesh, are specified so that the mesh becomes smaller around the circular hole and the discontinuous part of the domain where stress concentration occurs. In this process, the triangles formed by the mesh is regular as much as possible. Figure 12 shows mesh patterns generated by this method.

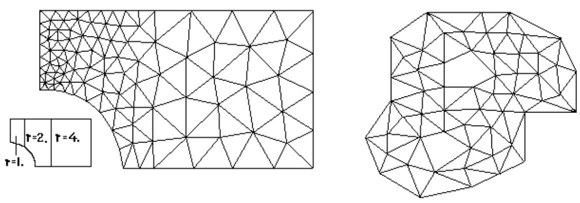
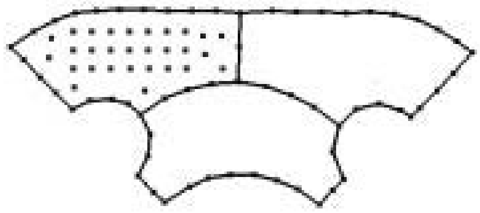
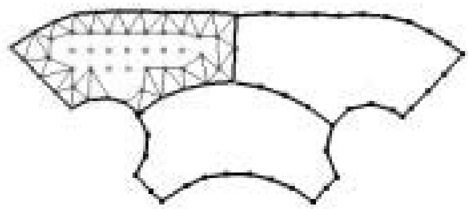


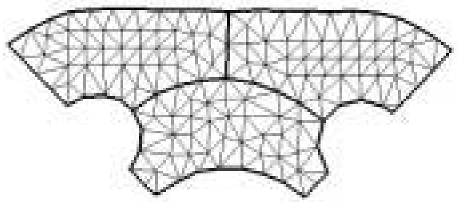
Figure 12: Mesh patterns generated by IMG method



(a) Nodes generated by random number



(b) process of generating mesh pattern



(c) Mesh pattern generated by IMG method

Figure 13: Example of Mesh Generator with uniform density(***r*** =17)

The process of generating and connecting nodes in order to form mesh in each sub-block is shown in figure 13.

3 Example of application

For the domain of the complicated shape[3], the treatment is carried out according to the algorithm described in **2**, and the result is shown in figures 14 to 16. It was possible to extract 48 block points and 72 line segments, and to construct domain of all 26 sub-blocks.

The line of extracted shape is segmented into segment units between block points, as shown in figure 14. Next, the result of approximating the shape of these line segments using Method II is shown in figure 15. Figure 16 shows mesh pattern generated with uniform density in each sub-block by IMG method.

It can be confirmed that to prepare the necessary data by converting the image information of the multiple connected domain of the complicated shape into numeric data, and constructing the shape of domain and sub-block domain by connecting the line segment, and approximating the line segment by polygonal line for the analysis is possible.

4 Conclusion

This pre-mesh generating system performs various image processing with respect to image data of the shape information of a sub-block for analysis object, and extracts the shape of the domain. Next, the shape of the domain is approximated by a sequence of points along the contour. Division of mesh is performed inside of a domain by the method of mesh generation the author has proposed previously, and input data of a mesh required for the analysis using the finite element method is created automatically. By using this system, a lot of input data required for analysis could always be prepared quickly and correctly, especially in a complex shaped or a multiple connected domain, using a personal computer and the practicality of finite element method is improved sufficiently.

5 References

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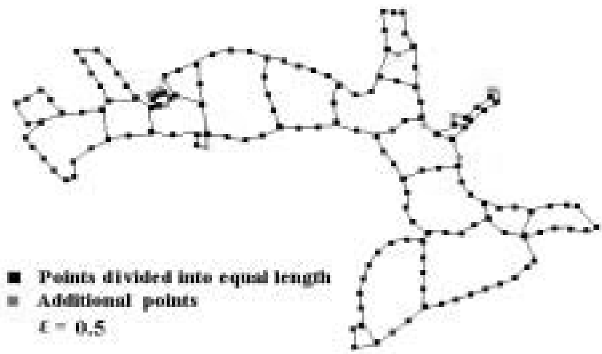

Figure 15: Selected points for defining the line segments by Method Ⅱ

(S_c=15,E_{max}=2.5)

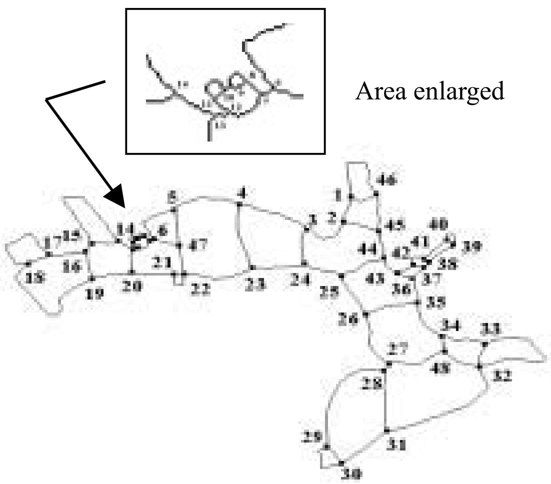

Figure 14: Construction of domains of the sub-block by connecting line segments with block points at the ends of the lines

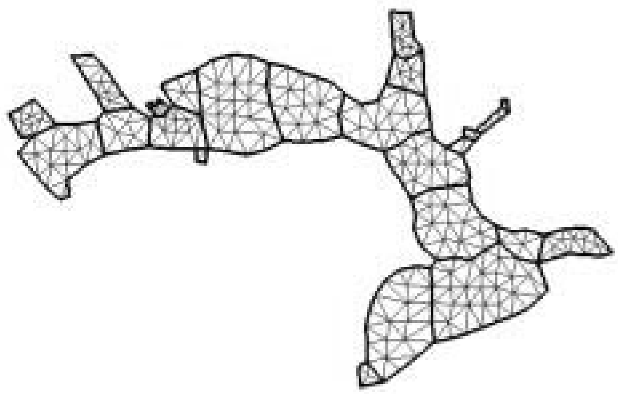

Figure 16: Mesh pattern generated by IGM method