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Modeling traffic flow at a single-lane urban roundabout

Ruili Wang, Heather J. Ruskin *

School of Computer Applications, Dublin City University, Dublin 9, Ireland

Abstract

In this paper, we propose a new model to study traffic flow at a single-lane urban roundabout, using a multi-state cellular automata (CA) ring under the offside-priority rule (by which a vehicle entering gives way to one already on the roundabout). Each vehicle entering the roundabout is randomly characterized by a predetermined exit with specified probability. Driver behavior at the roundabout entrance is randomly grouped into four categories based on space required to enter the roundabout. Three aspects of roundabout performance in particular have been studied. The first looks at overall *throughput* (the number of vehicles that navigate the roundabout in a given time). This is considered for different geometries, turning and arrival rates (vehicles arrive at random with a Poisson distribution, with parameter $\lambda \leqslant 0.5$ in general for free flow). The second investigates changes in queue length, delay time and vehicle density (ratio of the number vehicles to the number of cells) for an individual road. The third considers the impact of driver choices on throughput and operation of the roundabout. We find that throughput is influenced by the topology of the roundabout and turning rates, but only incidentally by size. Throughput reaches a maximum for critical arrival rate on one or more roads. Driver behavior has considerable impact on overall performance, with rapid congestion resulting from reckless choices. Vehicles drive on the left in Ireland, but rules are generally applicable.

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Keywords: Roundabout; Cellular automata; Throughput; Traffic flow; Modeling; Driver behavior

1. Introduction

Roundabouts are an important part of urban networks and transfer a complicated intersection into several simple T-intersections as well as reducing speeds.

Theoretical analysis of mobility and time delay in different traffic flows is an important issue in urban networks. Time taken to pass intersections and roundabouts contributes significantly to travel time and route choice [1].

Previous models of roundabout operations mostly focused on entry capacity models, where *entry capacity* (the number of vehicles that pass through an entrance per unit of time) was related to circulating flow of a single-lane roundabout (i.e. the total volume of traffic on the roundabout in a given period of time immediately prior to an entrance) [2–6].

2. Methodology

The *one-dimensional deterministic cellular automata model* (1DDCA), Yukawa [7] and Chopard [1, 8], is used to model a single-lane roundabout system.

* Corresponding author.
E-mail address: hruskin@compapp.dcu.ie (H.J. Ruskin).

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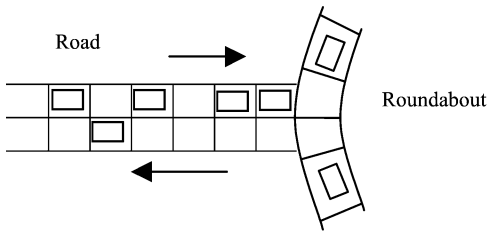

Fig. 1. A road and its entrance to a roundabout.

A multi-state CA ring is developed in order to characterize vehicle destinations. The state in each cell has three physical meanings. If zero, ($C = 0$), the cell is empty. If $C > 0$, the cell is occupied by a vehicle, where the value of C indicates how many cells it needs to traverse to arrive at the destined exit. The number of cells in the ring is determined by the real dimension of the roundabout and denoted N.

The model is related to the multi-speed models [9], which are critical to successful modeling of freeway traffic, but the latter have many features, which are superfluous for intersections [1] and roundabouts or to the representation of driver behaviour. Moreover, vehicle dynamics are often less important in simulating urban networks [1,10].

Fig. 1 illustrates a roundabout and entrance road for a multi-state 1DDCA ring and two 1DDCA directional traffic flows.

Under the offside-priority rule, drivers need to determine how much space on the roundabout is sufficient for them to drive to the required position and to gain enough speed so that no oncoming vehicle is obstructed.

We use the space available on a roundabout as the only parameter to describe driver behaviour here. **Optimum conditions** means that the space available on the roundabout is just enough for the vehicle to enter without interrupting flow. Free flow in 1DDCA model requires at least one free cell between each vehicle. Thus, at least three sequential vacant cells between roundabout vehicles are required for optimum entry.

Driver behaviour can be categorized as *conservative*, *rational*, *urgent* or *reckless* and considered as part of space criteria. A driver observing the optimum condition is **behaving rationally**, whereas **conservative behaviour** implies entry only when the space available $\geqslant 4$ cells. **Urgent behaviour** implies that a 2-cell space is acceptable. This action may cause the oncoming vehicle to pause for one time step. **Reckless behaviour** (down to a 1-cell space) may cause the oncoming vehicle to pause for two time steps and the entering vehicle to pause for one time step to avoid running into the vehicle in front. Blocking should not occur under strict operation of the offside-priority rule.

Clearly, the distribution of all drivers' behavior gives

$$P_{\text{co}} + P_{\text{ra}} + P_{\text{ur}} + P_{\text{re}} = 1, \tag{1}$$

where subscripts refer to conservative, rational, urgent and reckless behavior, respectively.

2.1. Update rules

2.1.1. Update rules for roads

If there is a vacant cell in front of the cell occupied by a vehicle, the vehicle will move forward one cell in the current time step. Otherwise, no movement is possible.

2.1.2. Entry rules for the roundabout

Simulation conditions for *rational behavior* are:

- Check the number of vacant cells (S) of the CA ring, in front and to the right of an entrance.
- If $S \geqslant 3$, the waiting vehicle at the entrance may enter the roundabout.
- If the first two cells are vacant and *the third one* is occupied by a vehicle exiting the roundabout, the waiting vehicle can also enter.

Similarly for other driver behavior. Uniform size and space of vehicles is assumed.

2.1.3. Predetermined exit for roundabout

Realistically, drivers make decisions on which exit is appropriate before entering. The approach used is to randomly assign each car a different number, which is equal to the number of the cells that the car needs to pass to arrive at its destined exit.

2.1.4. Up-date rules on the roundabout

If the state of cell n in time step t is denoted as C_n^t, the up-date rules are:

- If $C_n^t > 1$ and $C_{(n+1)}^t = 0$, then $C_{n+1}^{(t+1)} = C_n^t - 1$ and $C_n^{(t+1)} = 0$.

Table 1
N is even. Topology and turning rates are fixed

No.	Size (cells)	A1 to A2 (cells)	A2 to A3 (cells)	A3 to A4 (cells)	A4 to A1 (cells)	Throughput1 $\lambda = 0.15$	Throughput2 $\lambda = 0.20$	Throughput3 $\lambda = 0.25$	Throughput4 $\lambda = 0.30$
1	16	4	4	4	4	59,912	80,020	**99,410**	**99,906**
2	32	8	8	8	8	59,934	80,117	**99,415**	**99,925**
3	16	3	4	3	6	60,383	79,918	**99,396**	**99,876**
4	32	13	5	3	11	59,947	80,174	**99,496**	**99,813**
5	50	5	15	10	20	59,994	80,541	**99,402**	**99,891**

"A1 to A2" is the distance between the first and second entrance of the roundabout. Throughput1is related to all Poisson arrival rates $\lambda = 0.15$.

- If $C_n^t \geqslant 1$ and $C_{n+1}^{(t+1)} > 0$, then $C_n^{(t+1)} = C_{n.}^t$.
- If $C_n^t = 1$, then $C_n^{(t+1)} = 0$.

If cell n in time step t is "occupied" ($C_n^t > 1$), cell $n+1$ at t must be checked. For cell $n+1$ vacant, states of cell $n+1$ and n in time step $t+1$ change (i.e. $C_{n+1}^{(t+1)} = C_n^t - 1$ and $C_n^{(t+1)} = 0$). If cell $n+1$ is occupied, state of cell n in time $t+1$ does not change (i.e. $C_n^{(t+1)} = C_n^t$). As the car moves, its number will eventually become equal to one ($C_n^t = 1$), indicating the car will leave the roundabout in the next time step.

2.2. Theorems of density (ρ), throughput and size

Theorem 1. *If the number of cells in a roundabout is even, the ideal is all vehicles evenly distributed on the roundabout with optimum density* 0.5; *maximum throughput is then unaffected by size* ($= N$). *If* $\rho > 0.5$ *or* $\rho < 0.5$, *throughput < maximum.*

Theorem 2. *If* N *is odd* (*equal to* $(2n+1)$ *cells*), *optimum densities are* $n/(2n+1)$ *or* $(n+1)/(2n+1)$. *Both have the same maximum throughputs for given* N. *Throughput decreases if* $\rho < n/(2n+1)$ *or* $\rho > (n+1)/(2n+1)$. *Maximum throughput increases with the roundabout size* (N).

Proofs based on consideration of average speed and density, see [11].

3. Experimental results

Results for the length of each entrance road = 100 cells are shown in Table 1. Bold type indicates maximum queue or saturation on road(s). Experiments are carried out for 100,000 time steps. Optimum entry conditions are assumed.

3.1. Relationship between the size, shape and overall throughput

A four-arm roundabout (four entrances/exits) is considered for different N. Mean turning rates for left-turn, straight ahead and right-turn are 0.25, 0.5 and 0.25, respectively.

In Table 1, $N = 16, 32$ and 50 and distances between entrances are equal in the first two cases (equal spacing). Throughputs in each column (for all equal arrival rates) are similar, but change when arrival rates increase. Throughputs do not appear to depend on whether the spacing between the entrances is equal or unequal, as long as turning rates and topologies are the same. Similar results are found for other topologies, i.e. 3-arm roundabouts. The results indicate that overall throughput is not related to "even N" for roundabouts, given fixed topology, arrival rates, turning rates and optimum flow conditions.

In Table 2, non-equal spacing applies throughout. Throughputs increase with arrival rates for each N until road saturation is reached ($\lambda \geqslant 0.25$). As the roundabout size is increased, some fluctuation in throughput is observed but overall a slight increase is noted.

The experimental results broadly support the notion of an optimum density on the roundabout when optimum conditions apply. The size and geometry (spacing) of a roundabout have little direct influence on throughput for a single-lane roundabout for results of N chosen, since free flow conditions apply. Maximum throughput is obtained when N is even and optimum density can be achieved.

Table 2
N is odd. The topology and turning rates are fixed. Arrival rates are the same in each column

No.	Size (cells)	A1 to A2 (cells)	A2 to A3 (cells)	A3 to A4 (cells)	A4 to A1 (cells)	Throughput1 $\lambda = 0.15$	Throughput2 $\lambda = 0.20$	Throughput3 $\lambda = 0.25$	Throughput4 $\lambda = 0.30$
1	17	3	5	4	5	59,993	79,839	**96,533**	**96,705**
2	31	5	**7**	11	8	59,917	79,740	**97,638**	**97,614**
3	41	5	**17**	11	8	59,829	79,815	**97,994**	**97,914**
4	51	5	**27**	11	8	59,834	79,917	**98,089**	**98,293**

Table 3
Throughputs for changes in arrival rates (100,000 time step iterations)

$\lambda_4 =$ $\lambda_1 = \lambda_2 = \lambda_3$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55
0.05	20,065	25,913	29,878	34,988	40,235	44,951	50,195	54,913	**58,709**	**58,814**	**58,926**
0.10	35,062	39,159	44,993	50,123	54,751	60,698	65,337	**68,125**	**67,820**	**67,855**	**68,201**
0.15	50,234	55,614	59,765	64,941	70,015	74,776	**77,415**	**77,747**	**77,706**	**77,611**	**77,501**
0.20	65,349	70,714	74,937	80,127	85,129	**87,663**	**87,911**	**87,972**	**87,854**	**87,971**	**87,931**
0.25	***77,626***	***82,888***	***88,282***	***93,642***	***99,187***	**99,345**	**99,515**	**99,287**	**99,570**	**99,715**	**99,421**
0.30	**81,455**	**86,786**	**91,958**	**95,648**	**99,769**	**99,856**	**99,865**	**99,892**	**99,865**	**99,805**	**99,989**
0.35	**84,775**	**90,340**	**92,552**	**95,679**	**99,765**	**99,872**	**99,807**	**99,908**	**99,801**	**99,865**	**99,899**

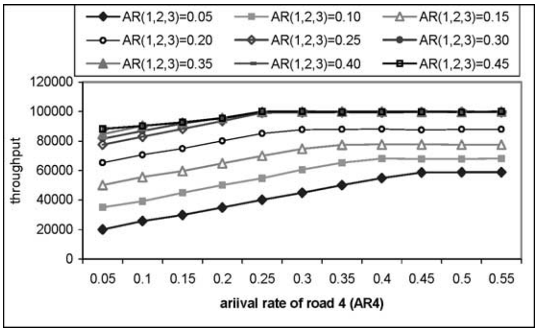

Fig. 2. Throughputs vs. arrival rates (AR).

3.2. *The relationship between throughput of a roundabout and arrival rates*

Table 3 and Fig. 2 show results for a series of experiments with arrival rates of three roads equal and arrival rate of road 4 ($AR_4 = \lambda_4$) varying from 0.05 to 0.55. For $\lambda_1 = \lambda_2 = \lambda_3 < 0.25$ (i.e. $AR_{(1,2,3)} < 0.25$), we find that throughput increases linearly as λ_4 increases, where no entrance road is saturated.

When $\lambda_4 \geqslant 0.40$ and $\lambda_1 = \lambda_2 = \lambda_3 = 0.10$, for example, road 4 is saturated and throughputs are constant (see table). The maximum throughput is achieved when road 4 saturates. For $\lambda_4 \geqslant$ **critical arrival rate** (CAR), saturation occurs on the entry road. Hence, $CAR_4 = 0.4$ for road 4 (indicated in shading in table). CAR varies with other three arrival rates.

If $\lambda = \lambda_1 = \lambda_2 = \lambda_3 < 0.25$,

$$\text{then } CAR_4 = 0.5 - \lambda, \tag{2}$$

$$\text{If } \lambda_1 = \lambda_2 = \lambda_3 \geqslant 0.25, \quad \text{then } CAR_4 = 0.25. \tag{3}$$

The throughputs reach a maximum rapidly and remain constant at this saturation level for all $\lambda \geqslant 0.25$. Defining the **effective throughput** as the throughput for no entrance road saturated, *maximum effective throughput* here is 96,080 in 100,000 time steps, achieved for $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0.24$. For arrival rates *not* equal, the effective throughput < 96,080.

3.3. *Throughput and turning rates*

Table 4 relates to cars driving on the left-hand side of the road (e.g., in the UK and Ireland). Results are based on a 32-cell 4-road-single-lane roundabout. Arrival rates are equal. The probabilities of right-turning rates (RTR) and left-turning rate are varied.

Turning rates have little impact on throughput for $\lambda < 0.25$ with traffic still in free flow. However, for entrance roads that are saturated, turning rates do affect throughputs. When $\lambda \geqslant 0.25$, 5% increase in RTR gives approximately 10% decrease in throughput.

Table 4
Throughputs of the roundabout for $\lambda = \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$

λ	Right-turning probability		
	0.15	0.25	0.35
0.15	60,024	59,765	60,002
0.20	80,515	80,127	80,031
0.25	100,416	**99,187**	**90,774**
0.30	**111,079**	**99,989**	**90,928**
0.35	**110,994**	**100,000**	**90,826**

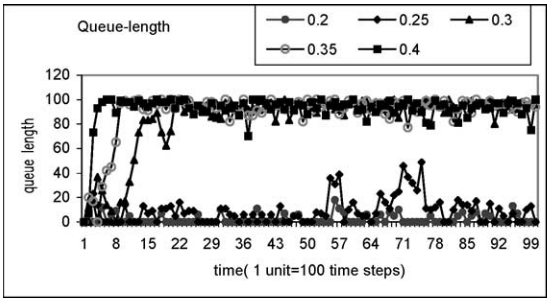

Fig. 3. Queue lengths on road 4 for various λ_4 (key); ($\lambda_1 = \lambda_2 = \lambda_3 = 0.20$).

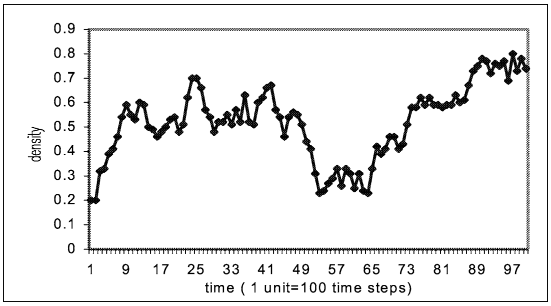

Fig. 4. Density road 2.

Table 5
Throughputs of roundabout for four examples of driver behavior. Arrival rates are the same for all roads

λ	Driver behaviour			
	$P_{co} = 1$	$P_{ra} = 1$	$P_{ur} = 1$	$P_{re} = 1$
0.10	40,011	39,996	39,985	40,105
0.15	60,277	60,234	60,233	**552**
0.20	**67,965**	79,516	79,810	**23**
0.25	**67,918**	**99,301**	**99,264**	**10**
0.30	**67,691**	**99,856**	**99,996**	**18**

3.4. Individual road performance—queue length

Queue length dependence on λ is clearly illustrated in Fig. 3 and corresponds closely to findings for throughput.

For $\lambda_1 = \lambda_2 = \lambda_3 = 0.20, \lambda_4 < \mathrm{CAR}_4$ $(= 0.30)$ queues are usually short. For $\lambda_4 \geqslant \mathrm{CAR}$, queue build-up is rapid. In $\sim$1000 time steps for $\lambda_4 = 0.35$ a queue of up to 100 cells results. For $\lambda_4 = 0.40$, between 500 to 800 time steps are needed to produce a similar length of queue.

3.5. Individual road performance—average densities on each road

Fig. 4 indicates car density for a given road (corresponding to queue length) when all $\lambda = 0.25$.

In general, queues forming for a value of ρ will be far below $\rho_{\max} = 0.5$(for free flow). Here, for example, a queue forms for $\rho = 0.23$, (as it is short, it does not stand out in the density measurement), while saturation occurs for $\rho \leqslant 0.8$. Similar results are found for other choices of arrival rates and turning rates, with queues forming for ρ in the range of 0.2 to 0.8 (similar to the findings for unsignalized intersections by Chopard [1]).

3.6. Driver behavior

In Table 5, all λ are equal. For low λ, throughputs are similar, but as arrival rates increase saturation occurs on the entry roads, so that for conservative driver behavior, throughput decreases. Little difference is observed between rational and urgent behavior, whereas reckless behavior results in congestion on the roundabout (gridlock) and throughput is drastically reduced. The final column of the table represents these extreme cases. Clearly, it is a simplification to denote drivers as collectively conservative, rational, urgent or reckless and a distribution would be more realistic.

3.7. Calibration and validation

Individual vehicle-vehicle interactions [12] are essentially confined to entrances. Probabilities of different driver behavior are arbitrarily chosen in our experiments, which would benefit from calibration on real data.

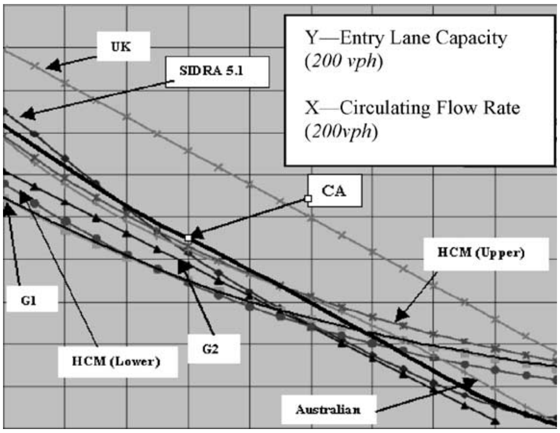

Fig. 5. Comparison of roundabout models: SIDRA5.1 HCM (upper and lower), UK, Australian and German (G1 and G2) vs. CA.

Lacking real data, we have calibrated our model by comparing it with previous models, which mainly analyze the relationship between entry capacity and circulating flow rate. For comparison, the circulation and entry situation have been simulated here.

Fig. 5 is reproduced from TPAU Oregon US [13] and compares our model result (CA) with SIDRA5.1, two Highway Capacity Manual methods (US), UK, Australian and two German (G1 and G2) methods.

According to recent investigations on critical gap and follow-up time [14], drivers use shorter critical gaps at high circulating rates due to the effect of longer delays, and use longer critical gaps when they do not need to wait so long to proceed. Similar criteria were used in the Australian capacity formula, which incorporated variations of critical gaps and follow-up times with different volumes of traffic in order to refine the gap-acceptance technique [15]. Arbitrarily changing the probability of conservative behavior from 0.5 to 0 (for circulating rates change from 0 to 1800 vph), based on Tian et al. [14], gives the curve CA, which appears to agree well with other models. Another criterion for change might be individual driver waiting time.

4. Summary

We have investigated a number of properties of single-lane roundabouts using a CA ring model. Principal findings are:

Roundabout size impacts little on throughput levels, given similar topology and arrival rates and turning rates fixed. A slight increase in throughput with size is observed when N is odd, where entry conditions are optimum but car density is not. Throughput levels in general depend on topology and the entrances are clearly bottlenecks to smooth operation.

In general, throughput increases linearly with arrival rate when no entrance road is in a saturated situation. It reaches a maximum when the arrival rate reaches a critical value on one or more roads (when saturation occurs). Throughput decreases as right-turning rate increases. Critical arrival rates depend on arrival rates (for all roads), on roundabout topology and on turning rates.

Speed of queue formation increases as arrival rates increase. Maximum queue length occurs within a few hundred time steps for arrival rates $\geqslant$ CAR.

Queue formation occurs at densities in the range of 0.2–0.8, which is similar to the result obtained by Chopard [1]. Queues form at densities well below the maximum for free flow ($= 0.5$).

Driver behavior impacts on overall roundabout performance measured by throughput figures, with reckless behavior leading rapidly to congestion.

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