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Effects of Passing Lanes on Highway Traffic Flow

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In this paper, the effects of passing lanes on traffic flow on a single-lane highway are investigated based on a cellular-automaton (CA) model proposed by Lee et al. (Phys. Rev. Lett. **92**, 238702 (2004)) with periodic boundary conditions. A new driving strategy for the diverging and interacting processes are proposed by considering dual-time-headways (an interaction headway and a safe headway) as the criteria for a vehicle to select a lane or change lane. The proposed dual-headway rule has a clear physical sense and can be calibrated in the real world. We can see that (i) At low- (or high-) density, there are no obvious increases of flux due to very weak (or very strong) interactions between vehicles. However, flux has an obvious increase at intermediate-density regions with the increase of the length of a passing lane; (ii) The space-time patterns before a passing lane are different from that after a passing lane, which is caused from vehicular reconfigurations on a passing zone; (iii) Passing lanes can merge several narrow jams into one or few wide jams when increasing the length of passing lanes; (iv) If vehicles are allowed to travel with even higher speed, traffic can be further improved.

Keywords: Passing lanes; cellular automata; highway traffic flow; traffic flow modeling.

1. Introduction

A variety of microscopic and macroscopic models (e.g., in Refs. [1,2,3,4,5,6,7,8]) have been presented for modelling highway traffic and city traffic[3,?]in recent years. Among them, cellular automata (CA) traffic models have received much attention as CA are dynamical systems, in which space, time, and variables are discrete [9] and can be used for large-scale real-time traffic simulations [10].

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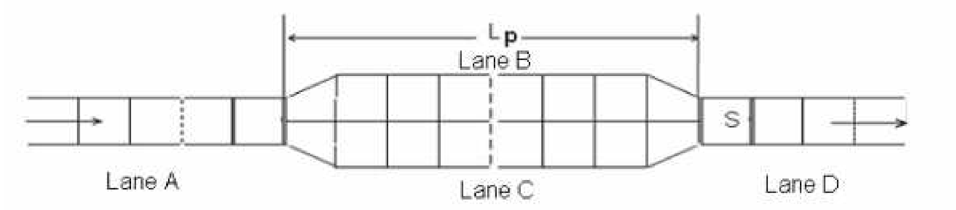

Fig. 1. Schematic diagram of a combined traffic system. Lanes A and D - single lanes, lane B - main lane (or left lane) and lane C - passing lane (or right lane). *Lp* denotes the length of a passing zone.

In order to model traffic flow realistically, single-lane and multi-lane CA models, e.g., in Refs. [11,12,13,14,15,16,17,18,19,22], have been developed. Unfortunately, multi-lane highways are not always available in countries such as Australia, Finland, New Zealand, Ireland, Japan and Sweden due to financial and/or geographic reasons. As a consequence, passing lanes have been constructed as an alternative traffic facility for the improvement of traffic conditions on single-lane highways.

In a single-lane highway system, overtaking using an opposite lane is sometimes forbidden due to geographic and/or safety reasons. A passing lane provides an overtaking opportunity for fast vehicles in a single-lane traffic system. This kind of traffic system can be viewed as a combination of single- and two-lane systems (see Fig. 1). When vehicles approach a passing zone (where a passing lane is facilitated), fast vehicles can overtake slow vehicles in front and thus escape from platoons. A platoon is a number of vehicles travelling together as a group, either voluntarily or involuntarily because of geometrics of roads, speed differences of vehicles, signal control or other factors.

In general, traffic flow going through a passing zone (see Fig. 1) involves three processes: the diverging process (one traffic flow diverging into two flows), interacting process (interacting between the two flows), and merging process (the two flows merging into one). The process when traffic flow on a single-lane highway diverges onto two lanes (diverging process) has some similarity to an off-ramp system. Investigation on off-ramp systems can be found in Refs. [20,21]. However, the diverging process and an off-ramp system are different in nature since in an off-ramp system, vehicles on the off-ramp do not interact (e.g., lane change) with that on the main lane as they have different driving directions, i.e., an off-ramp leads to an exit. Also, when vehicles leave a passing zone, they are merged into a single-lane highway. We can see some similarity between the merging process at the end of a passing zone and the merging process in an on-ramp system. However, the yield rules are different (the yield rule in the merging process can be found in Section 3). In the passing zone, all vehicles should not use the passing lane, unless it is necessary for passing. This rule will be used in Section 3.

Individual vehicles that pass through a passing zone may be involved in the processes of lane selecting, overtaking (lane changing) and merging. Thus, traffic flow dynamics in a passing zone are more complicated than a simple diverging process in an off-ramp system or a merging process in an on-ramp system. Therefore, we propose a new driving strategy for the diverging and interacting processes, which will consider two time headways (an interaction headway and safe headway) as the criteria for lane-changing and lane-selecting.

In this paper, we investigate the effects of passing lanes on traffic flow by finding the properties of traffic flow using some standard techniques, such as the fundamental diagram, space-time patterns, space-speed patterns and mean speed patterns. Simulation results have shown that the length of a passing lane, density, and the maximum speed on passing lanes can affect traffic flow.

This paper is organized as follows. In Section 2, CA traffic models on single-lane and two-lane highways are briefly reviewed. In Section 3, three processes: diverging, interacting and merging are described. Also, the corresponding driving strategies, based on the interaction headway and safe headway, are presented, which suggests that time headways are more appropriate for the description of diverging and interacting processes, rather than distances as used in most existing multi-lane CA models. In Section 4, the simulation results are discussed and some findings are reported. Finally, a summary is given in Section 5.

2. Background

In the real world, traffic flow is normally a mix of many kinds of vehicles, e.g., cars, buses, vans and trucks. These vehicles with different dimensional and dynamic properties form heterogeneous traffic flow. Empirical observations and simulation results have shown that: (i) Slow vehicles dominate heterogeneous traffic flow and can lead to platoon formations at low densities in single-lane traffic [23,24,25,26,27]; and (ii) The maximal flux decreases when increasing the number of long vehicles [28]. Treiber and Helbing [29] suggest that heterogeneous traffic flow can be one of the reasons for widely scattered data points over a two-dimensional plane in congested states. They reported that their simulation results were in good agreement with Dutch highway data.

Dynamic properties of heterogeneous single-lane highway traffic flow have been further studied by Wang et al. [31], under the framework of three phase traffic flow theory [32]. The Cross Correlation Function (CCF) analysis has shown that heterogeneous flow has almost the same strong coupling as homogeneous flow [31]. In other words, flux and density are highly correlated in free flow and jams, while they are almost independent in synchronized flow.

When single-lane traffic is extended to two- or multi-lane traffic, more attention has been attracted to the study of lane-changing behaviour. Based on symmetric (all lanes can be used for vehicles to overtake) and asymmetric (only specified lanes can be used to overtake) lane-changing rules, a great number of two- or multi-lane CA models have been proposed. Many interesting phenomena have been reported and/or reproduced. For instance, lane-usage inversion was investigated in Ref.[33] and the effects of lane changing with two types of vehicles are studied in Ref.[34]. Density inversion that occurred below the maximum flow density, is reproduced in Ref. [26]. Knospe et al. [27] proposed a new two-lane model which takes the anticipation effects (i.e., drivers observe their predecessors' speeds in following time steps) into account. Moussa and Daoudia [14] compared symmetric and asymmetric lane-changing rules in a car-truck system. Their simulation results confirmed that slow vehicles dominate the behaviour of two-lane systems even with a small number of slow vehicles, and showed that the flux in asymmetric lane-changing rules always exceeds that in symmetric lane-changing rules. Jia et al. [16] investigated the honk effects (a vehicle may honk the horn to let the preceding vehicle make way for it) in two-lane traffic. Aggressive lane-changing behaviour of fast vehicles has been modelled in Ref.[18]. It is shown that aggressive driving behaviour can improve traffic flow in mixed traffic in the intermediate density range.

Phase transition from free flow to synchronized flow has also been studied in multilane traffic flow. Ref. [35] introduced variable h, called order parameter, to describe the internal correlation between vehicles along different lanes. A large value of the order parameter characterises synchronized flow, whereas free flow and jams match its small values because of weak mutual interactions (i.e., lane changing) between vehicles. Recently, Kerner and Klenov [36] extended their KKW model [15] to a two-lane heterogeneous traffic. The extended KKW model can reproduce the vehicle lane separation effect (fast vehicles use mostly one lane, whereas other vehicles use the other lane) in free flow.

In this paper, we use the MR (mechanical restriction) model, proposed by Lee et al. [37], to simulate traffic flow at a passing zone with periodic boundary conditions. The major assumption of the MR model is that a following vehicle should always prepare for the worst case, namely, the preceding one may brake suddenly at any time step. Based on this assumption, mechanical restriction (limited deceleration capability) and human overreaction (a driver's behaviour may be biased according to the local traffic condition) are used to explain congested traffic states. Their model can satisfactorily reproduce three different traffic phases, i.e., free flow, synchronized flow, and wide moving jams and the transition from synchronized flow to jams.

3. Methodology

In a passing zone, traffic flow may involve three processes: diverging, interacting (lane-changing), and merging. The diverging process corresponds to vehicles entering a passing zone from a single-lane zone. The interacting process describes lane-changing behaviour in a passing zone. The merging process corresponds to vehicles exiting from a passing zone.

3.1. *Diverging process*

When traffic flow is entering a passing zone, it will diverge into two flows: one goes to a main lane; the other one goes to a passing lane. When a vehicle enters the passing zone, the driver needs to decide which lane to choose. This decision will be made based on the type and the dynamic properties (e.g., the positions and speeds of vehicles) of the last vehicles on a main lane and passing lane. The basic policy is to choose a lane with less interaction with the preceding vehicles. In other words, a fast vehicle will choose a lane in which it can drive at a higher speed.

The study in Ref. [38] has shown that a better way to describe the conditions of interaction between vehicles is using a tempo-spatial parameter, such as time headway, rather than a spatial one, such as distance. Thus, in this paper, we suggest a *dual-(time-)headway* rule, i.e., using two headways; the interaction headway (H) and safe headway (h), to describe the driver behaviour of both selecting lanes and changing lanes (see Section 3.2). Time headway is normally defined as the time, in seconds, expressed by the distance between two successive vehicles divided by the speed of the following vehicle.

Interaction headway H is used to represent the range of interaction between vehicles. If a time headway is longer than H, it means that there will be no interaction between successive vehicles. In other words, the speed of the following vehicle will not be affected by the leading one that is out of interaction range H. Here, we adopt $H = 6$ as suggested in Ref.[38]. Safe headway, $h = 2$, is the minimum safe headway for a vehicle to follow its leading vehicle, recommended in many countries including New Zealand [39].

According to the road code of passing lanes in New Zealand, "keep left (i.e., on the main lane) unless passing", a driving strategy for vehicles in a diverging process can be described as follows:

- Case 1: If an entering vehicle is a slow vehicle (e.g., a truck), it will directly enter the main lane. Theoretically, no slow vehicles need to overtake other vehicles according to the road code;

- Case 2: If an entering vehicle is a fast vehicle (e.g., a car), it will need to check the type of the preceding vehicle and to determine the time headway to its predecessor on the main lane first and may need to check that on the passing lane. Then, it can decide to enter a proper lane. Let $g_m^n(t)$ and $g_p^n(t)$ denote the gaps (i.e., the numbers of free cells) of vehicle n to the last vehicles on the main and passing lanes at time step t. $h_m^n(t)$ and $h_p^n(t)$ denote the corresponding time headways of $g_m^n(t)$ and $g_p^n(t)$. We have $h_m^n(t) = g_m^n(t)/v_n(t)$ and $h_p^n(t) = g_p^n(t)/v_n(t)$, where $v_n(t)$ denotes the speed of vehicle n.
  - Subcase 1: If $h_m^n(t) = H$, the entering vehicle will enter the main lane. This is because that the preceding vehicle on the main lane is out of the interaction range. In other words, the entering vehicle does not need to consider lane change for a certain period of time even though the preceding vehicle is a slow one.
  - Subcase 2: If $H > h_m^n(t) \geq h$, we have (i) If the last vehicle on the

main lane is a slow vehicle (e.g., a truck) and $h_p^n(t) \geq h$, the entering vehicle will enter the passing lane in order to avoid being hindered by the preceding vehicle on the main lane; (ii) Otherwise, the entering vehicle will enter the main lane. Note that the vehicle will enter the main lane even if the last vehicle on the main lane is a slow vehicle provided $h_p^n(t) < h$. If $h_m^n(t) < h$, we have (i) If $h_p^n(t) = h$, the entering vehicle will enter the passing lane; (ii) If $h_p^n(t) < h$, the vehicle will choose the lane which has a longer time headway, i.e., comparing $h_m^n(t)$ and $h_p^n(t)$.

3.2. *Interacting*

Previous lane-changing rules are normally based on distance estimation in most existing multi-lane CA traffic models (e.g.,[14,15,16,18,26,27,33,34,42]), which are considered to be feasible for vehicles if there are sufficient spaces (i.e., distances) ahead and behind in the target lane so that vehicles can safely change to the target lane.

As argued in Ref. [38], a tempo-spatial parameter, such as time headway, may be better than a spatial one, such as distance, in considering the interactions between vehicles for lane-changing. When a driver changes lane, he/she will consider not only the distances, but also the relative speeds to other vehicles. Based on this argument, a new asymmetric lane-changing strategy, the dual-headway rule (as explained in last section) for fast vehicles to change lane in a passing zone is presented. The updated rules have be developed based on the road code. The road code is described as follows: (i) All vehicles should use a main lane if they can; (ii) Slow vehicles (e.g., trucks) can only drive on the main lane; and (iii) Fast vehicles (e.g., cars) can drive on both main and passing lanes. Two sets of lane-changing rules (from a main lane to a passing lane and from a passing lane to a main lane) are given as follows.

- Case 1: If fast vehicle n is on a main lane and its preceding vehicle is a slow vehicle, and the time headway is smaller than H, vehicle n will change lanes when the following conditions are met: $HF_p = h$ and $HB_p = h$. Here, HF_p and HB_p are the time headways of the vehicle on the passing lane in front and the vehicle behind vehicle n on the passing lane.

- Case 2: Fast vehicle n will change to the main lane when the following conditions are met:
  - Subcase 1: $HF_m = H$ (if the preceding vehicle on the main lane is a slow vehicle) and $HB_m = h$.
  - Subcase 2: $HF_m = h$ (if the preceding vehicle on the main lane is a fast vehicle) and $HB_m = h$. Here, HF_m and HB_m are the time headways of the vehicle on the passing lane in front and the vehicle behind vehicle n on the main lane.

3.3. *Merging process*

At the end of a passing zone, traffic flow on the main lane and traffic flow on the passing lane will merge into one flow. Vehicles from both the main lane and passing lane will eventually merge into one lane. According to the yield rule observed in counties such as the UK, Australia and New Zealand, vehicles driving on the left lane (main lane) need to give way to vehicles on the right lane (passing lane). Therefore, we propose the following driving strategy for vehicles in a merging process.

- Case 1: If a vehicle is driving on the passing lane, it will enter the single-lane highway straight away.
- Case 2: If a vehicle is driving on the main lane, it needs to check if it needs to give way to the vehicle on the passing lane. t^m and t^p denote the required time to pass cell S (see Fig. 1) for the first vehicle on the main and passing lanes.
  - Subcase 1: If $t^m \neq t^p$, the vehicle which needs less time to arrive at cell S can enter the single lane (lane D) first.
  - Subcase 2: If $t^m = t^p$, by dropping its speed, the vehicle on the main lane needs to give way to the vehicle on the passing lane.

4. Simulation results and discussion

This paper considers two types of vehicles, fast vehicles such as cars and slow vehicles such as trucks. $v^f_{max} = 20$ and $v^s_{max} = 17$ correspond to the maximum speeds of fast and slow vehicles. $R_f : R_s = 0.9 : 0.1$ denotes the ratio of fast vehicles and slow vehicles (e.g., cars and trucks) to all vehicles. Each cell corresponds to 1.5 m in a real road. One time step corresponds to one second. The length of fast vehicles like cars is set to be 5 cells and the length of slow vehicles like trucks are set to be 10 cells. The values of interaction headway and safe headway are set to be 6 and 2 time steps (i.e., seconds) respectively. Other parameters are set to be $a = 1, D = 2, v_{fast} = 19, t_{safe} = 3, g_{add} = 4, p_d = 0.11, p_0 = 0.32$, and $v_{slow} = 5$ as in the Ref. [37]. Each simulation is carried out for 3,600 time steps after the relaxation of 20,000 time steps. The road is divided into 10,000 cells which are empty or occupied by vehicles. The length of a passing lane is assumed to be 1,000 cells from cell 8,001 to cell 9,000 of these 10,000 cells.

Ref. [37] has mentioned that the collision of vehicles can be occasionally found near an on-ramp in the MR model. We have also observed this phenomenon (i.e., collision of vehicles) in a lane-changing process. In order to avoid collisions, we extend the single deceleration capability in the MR model to two different deceleration capabilities: (i) $D_1 = 2$, the deceleration capability for normal conditions, and (ii) $D_{max} = 4$, the maximum deceleration capability for emergent conditions, which are included in the lane-changing and merging processes. This extension has basically solved the contradiction between the limited deceleration capability and avoidance

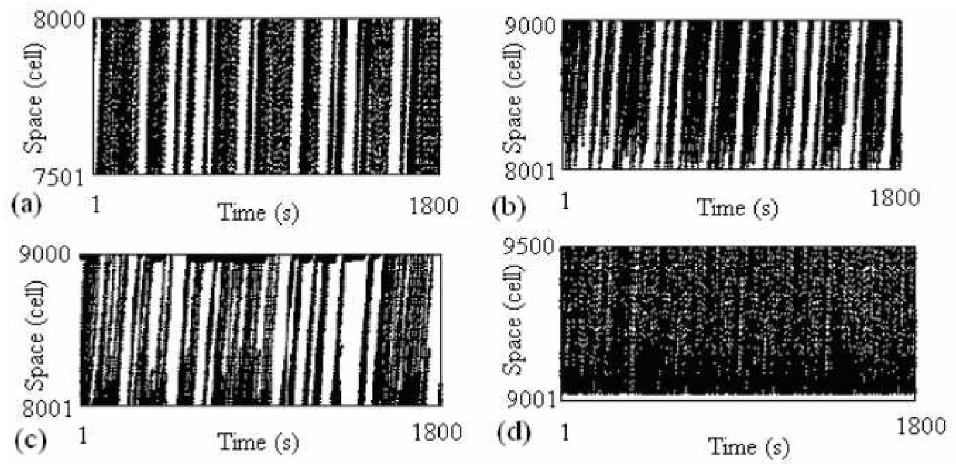

Fig. 2. Space-time patterns at $\rho = 0.14$. (a) the last 500 cells before entering a passing zone, (b) the passing lane, (c) the main lane, and (d) the first 500 cells after passing.

collision. $D_{max} = 4$ corresponds to 6 m/s^2 in the real world, which is close to the recommended maximum deceleration rate 6.4 m/s^2 [30].

4.1. *Effects of a passing lane*

In order to find out the effects of passing lanes on traffic, we will investigate speed evolution over time in the following places: the last 500 cells before entering the passing zone (i.e. cell 7,501 to cell 8,000), the whole length in the passing zone (from cell 8,001 to cell 9,000), and the first 500 cells after a passing zone (from cell 9,001 to cell 9,500). Also, we show the fundamental diagrams (flux versus density) with and without passing zone(s) to measure the performance of passing zone(s). The periodic boundary condition has been used in this paper.

Figs. 2 and 3 show the space-time and speed-space patterns on two areas of a single lane (500 cells before and after the passing zone) and a passing zone at density $\rho = 0.14$. We can see: (i) Vehicles travel at platoons [Figs. 2(a)] (ii) Slow vehicles dominate the mixed traffic by their maximum speed ($v^s_{max} = 17$) [Fig. 3(a)] before vehicles enter the passing zone. These phenomena in our simulation of mixed traffic flow in the single-lane highway areas are consistent with the results in Refs. [23,24,25,26,27] and empirical findings in Ref. [41].

When vehicles drive in a passing zone, speed on the passing lane is mainly dominated by one value: 20, see Fig. 3(b). This implies that fast vehicles drive at their desired speeds in the passing lane. Whereas two values (20 and 17) dominate the mixed traffic on the main lane [Fig. 3(c)]. This means that a fast vehicle usually either drives at its desired speed or adjusts its speed to follow the slow vehicle in front (a car-following flow) if no appropriate lane-changing chances exist at present. Clearly, slow vehicles on the main lane will drive at their desired speed, 17.

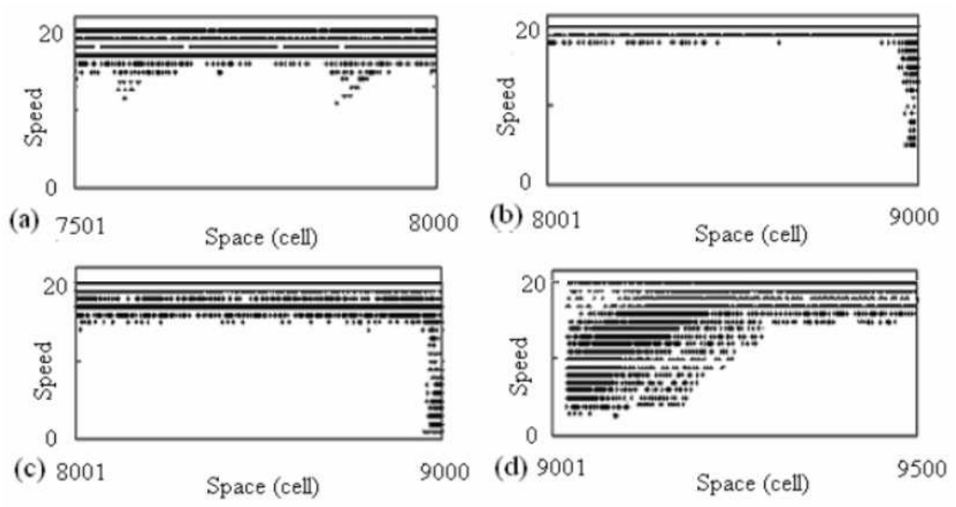

Fig. 3. Plots of speed against space. (a) the last 500 cells before entering a passing zone, (b) the passing lane, (c) the main lane, and (d) the first 500 cells after passing.

At the downstream end of a passing zone, vehicles on the main and passing lanes are involved in a merging process. As a consequence, vehicles interact strongly and a large decrease of speed can be found on the main lane, which results in the occurrence of jams. This process is captured more clearly on the upside of Fig. 2(c) and the right of Fig. 3(c). Fig. 3(d) illustrates the speed profile after passing. It is found that a great number of vehicles have slow speeds at the beginning of the single lane after passing. It means that large fluctuations of speeds are caused by the merging process. Moreover, the figure also shows only a few vehicles with higher speeds at the beginning of the single lane after passing [see the top-left corner of Fig. 3(d)].

Note that the phenomena of lane-selecting can be obviously observed at the beginning of the passing zone as numerous fast vehicles will choose the passing lane [Fig. 2(b)]. However, the phenomenon of fast vehicles changing back to the main lane is not obvious. We argue that our dual-headway rule has better reflected the real situation that fast vehicles will stay on the passing lane unless the density of slow vehicles on the main lane is very low. The effects of different time-headway requirements and lane-changing probability on lane changing have not been discussed in this paper as we will focus on these issues in another study.

In the first 500 cells after a passing zone, disordered space-time and speed-space patterns can be found at the bottom of Fig. 2(d) and on the left side of Fig. 3(d) where the speeds of vehicles fluctuates abruptly. With the increase of the distance from the merging zone, fluctuations of the speeds tend to smooth. Also, the domination value, 17, is finally recovered on the right side of Fig. 3(d).

From our simulations, we also find that the space-time patterns are different

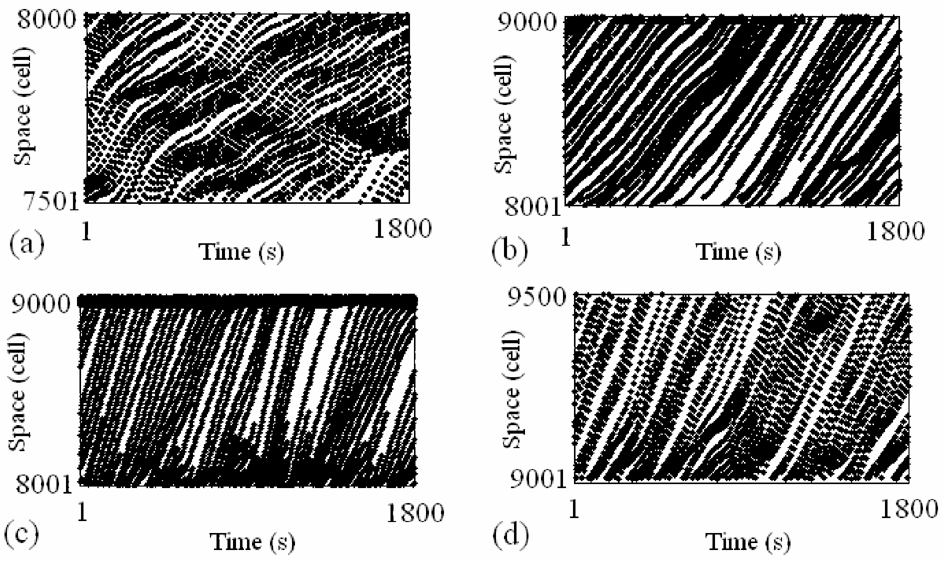

Fig. 4. Space-time patterns at density $\rho = 0.24$. (a) the last 500 cells before entering a passing zone, (b) the passing lane, (c) the main lane, and (d) the first 500 cells after passing.

when density is different. Comparing Fig. 4 ($\rho = 0.24$) with Fig. 2 ($\rho = 0.14$), there are several differences, in particular, (i) Free flow and synchronised flow coexist in Figs. 4(a) and (d), while only free flow exists in Figs. 2(a) and (d). This is consistent with empirical observations in Ref.[32]; (ii) Jams occupy more space in the downstream end of the main lane in Fig. 4(c) than that in Fig. 2(c); (iii) Small jams appear in the downstream end of the passing lane in Fig. 4(b), while no jams occur in Fig. 2(b).

We next investigate the effects in the fundamental diagram by comparing the following two cases: (i) single-lane heterogeneous (mixed) flow with a passing lane, and (ii) single-lane heterogeneous flow without a passing lane. In Fig. 5, it can be seen that density is divided into three different regimes. When density is low ($\rho < \rho_1$), a passing lane has little effect on traffic in terms of flux due to sparse vehicles in the system. When density is high ($\rho > \rho_2$), there is no obvious increase in flux either. This is because the number of vehicles that pass through the passing lane will increase, but the number of vehicles that pass through the main lane decreases due to the yield rule. The merging process becomes a bottleneck. Thus, the total number of vehicles that pass the passing zone has no substantial increase. Between the two densities, ρ_1 and ρ_2, the maximum flux in case 1 (with passing lanes) is larger than that in case 2 (without passing lanes) by approximately 6%.

The fundamental diagram implies that the introduction of a passing lane can improve traffic flow to some extend. The natural questions will arise: Is the length of a passing lane relevant to the flux? How does the length of a passing lane affect traffic flow? We will study these issues in the next subsection.

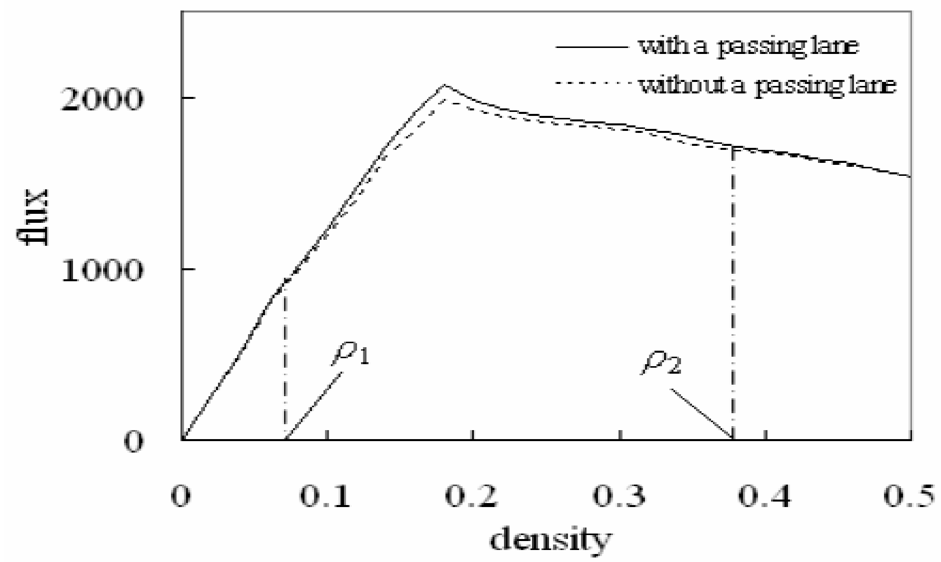

Fig. 5. Fundamental diagram of traffic flow with or without a passing lane under periodic boundary conditions. The length of the passing lane is 1,000 cells.

4.2. *Effects of lengths of passing lanes*

In this section, we first investigate the fundamental diagram under different lengths of passing lanes then discuss the relationship between the increase of flux and the increase of construction cost. Finally we study velocity evolution over time under different densities.

There are two extreme situations for a single-lane highway system. We first consider an extreme situation, no passing lane. In this case, the system becomes a normal single lane system. As we know, in such case, traffic flow is dominated by slow vehicles as indicated in Refs. [26,27,41]. With regard to the other extreme situation, a single-lane highway is extended to become a two-lane highway. It is out of the scope of this paper.

We first examine the effects of longer passing lanes (L_p = 1,000, 3,000, 5,000 and 8,000) when the length of the system is 10,000. It can be seen in Fig. 6: (i) When the density is very low or high, the length of a passing lane has little effect on traffic flux; (ii) The flux has obvious increases with the increase of the length of a passing lane at intermediate density. For intermediate densities, according to the lane-changing rules in Section 3, lots of fast vehicles will select passing lanes to drive on, so that they can drive with the maximum speed v^p_{max}, while vehicles travel with several platoons on the single lane. Thus, passing lanes can be effectively utilised. However, when density increases further, jams occur on the road sections with single lane. The effects of passing lanes become smaller.

We next investigate the effects of shorter passing lanes (L_p = 100, 300, 500, 700

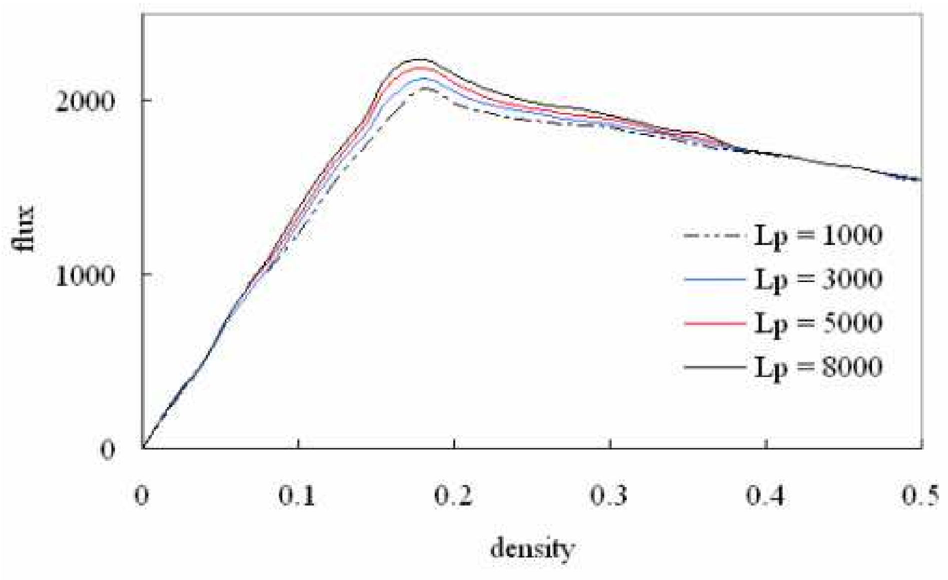

Fig. 6. (Colour online) Fundamental diagram with longer passing lanes.

and 900) with the length of the system 10,000. When the length of a passing lane is very short (i.e., $L_p = 100$, 300 and 500), the flux is almost equal to that without passing lane (i.e., $L_p = 0$), see Fig. 7. In such short passing lanes, fewer vehicles have opportunities for overtaking. Thus, there is no obvious difference between flux-density curves. When length $L_p = 500$, the flux has shown some increase and is almost equal to that with the passing-lane length $L_p = 1,000$ in Fig. 7.

Fig. 8 shows the relationship between the increased maximum flux rate and the increased construction cost rate under shorter passing lanes. We believe that it is useful to evaluate this relationship when traffic engineers consider adding a passing lane in a single-lane system. A single-lane system without a passing lane is used as a base. We define $r_{flux} = (f^{Lp}_{max} - f_{max})/f_{max}$ as the increase rate of the maximum flux rate; $f^{L}_{max}p$ denotes the maximum flux for different L_p, and f_{max} denotes the maximum flux when $L_p = 0$. We arbitrarily assume that the construction cost increases linearly with the increase of the length of a passing lane. That is, $r_{cost} = L_p/L$, where r_{cost} denotes the increase rate of construction cost; L_p and L denote lengths of a passing lane and the system respectively. When $L_p = 0, r_{flux} = 0$ and $r_{cost} = 0$, i.e., no extra construction cost for a single-lane system without a passing lane. When $L_p = 10,000, r_{flux} = 1$ and $r_{cost} = 1$, i.e., a single-lane system is completely extended into a two-lane system. Thus, the maximum flux increases by one time as well as the construction cost.

We can see that the increase of r_{flux} is not linear (see Fig. 7). When $r_{cost} < r_1$ or $r_{cost} > r_2$ (see the inset), r_{flux} increases slowly. This means that it is not economical to consider a passing lane in these regions of the construction cost rates. When

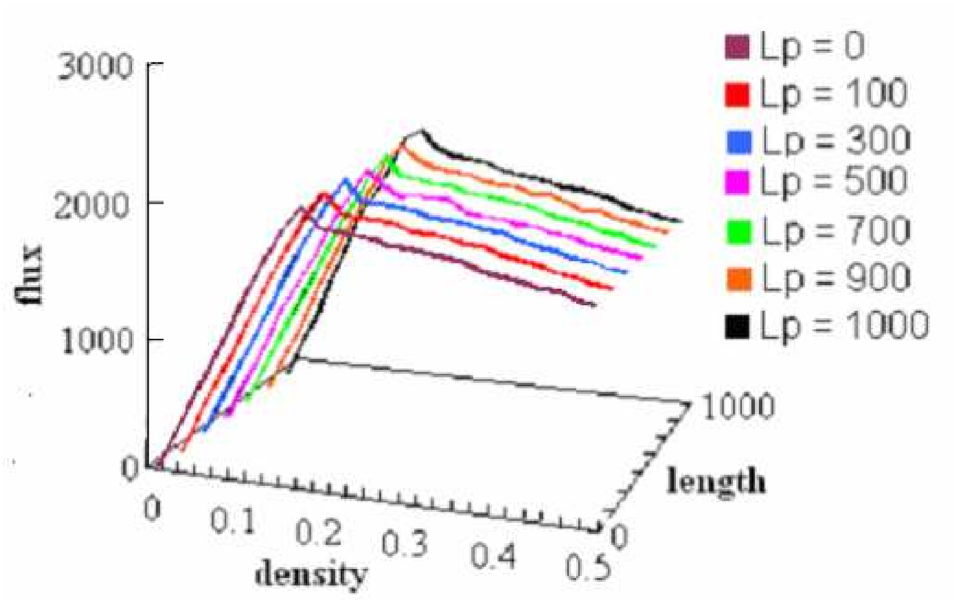

Fig. 7. (Colour online) Fundamental diagram with longer passing lanes.

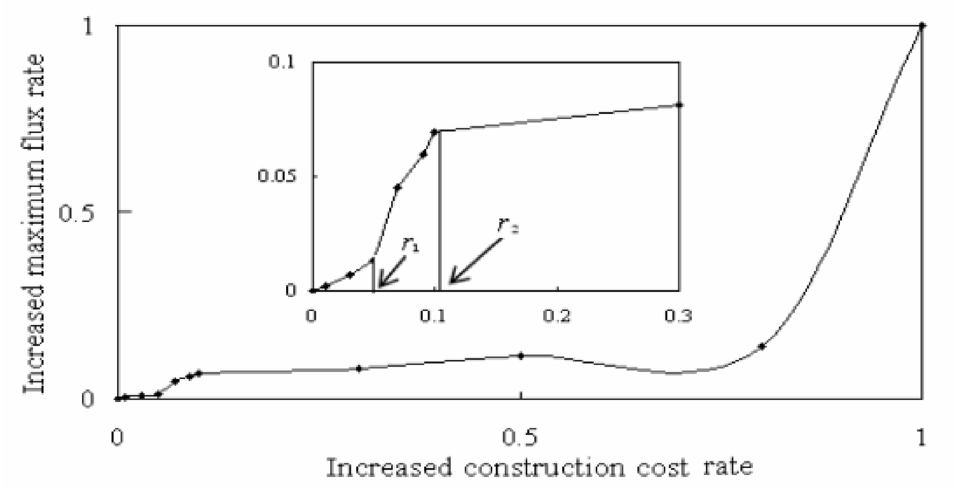

Fig. 8. Comparison of increased maximum flux rate and approximately required construction cost rate under the length of the system L = 10,000 cells. The inset zooms into the initial transition region.

r_{cost} is between r_1 and r_2 (see the inset), r_{flux} has significant increase. This implies that the length between 500 and 1,000 cells (corresponding to 750m and 1,500 m respectively) may be appropriate for a passing lane as there is a higher ratio between r_{flux} and r_{cost}. This is consistent with Refs. [44,45,46] where the suggested optimum length of a passing lane is between 800 m and 1,500 m.

In general, we can see the increase of flux by increasing the lengths of passing lanes is not economical as the ratio between r_{flux} and r_{cost} is smaller than 1 when $L_p > 0$ and $L_p < 10,000$. If the lengths of passing lanes are too short, fast vehicles do not have sufficient time to overtake slow vehicle(s) in front within a passing zone. If they are too long, vehicles may not need such long passing zones and construction costs will increase. Therefore, it is not surprising that the main purpose of passing lanes is to provide opportunities for overtaking. Increasing flux is not the main purpose.

Fig. 9 shows mean speed evolution over time under different lengths of passing lanes at different densities. In Fig. 9 (a), the mean speed increases with the increase of the lengths of passing lanes at very low density. When length $L_p = 8,000$, the mean speed approaches the maximum speed. This means that fast vehicles can get a higher speed on the passing lane. Note this increase of mean velocity does not mean the increase of flow rate because the mean velocity in the single lane is still determined by slow vehicles. We also found that platoons are disturbed and shortened when increasing the length of a passing lane, which is consistent with the results in the study in Ref. [43].

When traffic is in intermediate density, it can be seen that the mean speed still increases with the increase of the lengths of passing lanes. However the mean speed is lower than that in very low density. This is because when the density is lower, fast vehicles have more opportunities to drive at desired speeds. In this case, although the mean velocity in the single lane is still determined by the slow vehicles, the flow rate increases because the density in the single lane section increases notably.

If the density is high, it is found that mean speed decreases with the increase of the lengths of passing lanes. Besides the reasons mentioned above, a jam is formed and fixed at the forepart of passing lanes. When the length of a passing lane is increased, lots of vehicles will drive on the passing lane. At the merging zone, vehicles on the main lane will give way to those on the passing lane according to the yield rule. Thus, vehicles on the main lane have to slow down and even stop to let vehicles on the passing lane go first, which leads to the decrease of speed of vehicles on the main lane.

4.3. *Effects of the maximum speed of fast vehicles*

This section will investigate such a condition: How traffic can be affected if the maximum speed in a passing lane is larger than 20 cells? Normally, the maximum speed of a fast vehicle in a passing lane is the same as that in a main lane, i.e., $v^f_{max} = 20$ cells. If we assume the maximum speed of a fast vehicle in a passing lane as v^p_{max}, while the maximum speeds of vehicles in a main lane is unchanged (i.e., 20 cells for a car and 17 cells for a truck), we can see that Fig. 10 shows the flux-density relationship under the different values of v^p_{max}. At low density, the flux increases linearly with density. Therefore, different values of v^p_{max} do not lead to

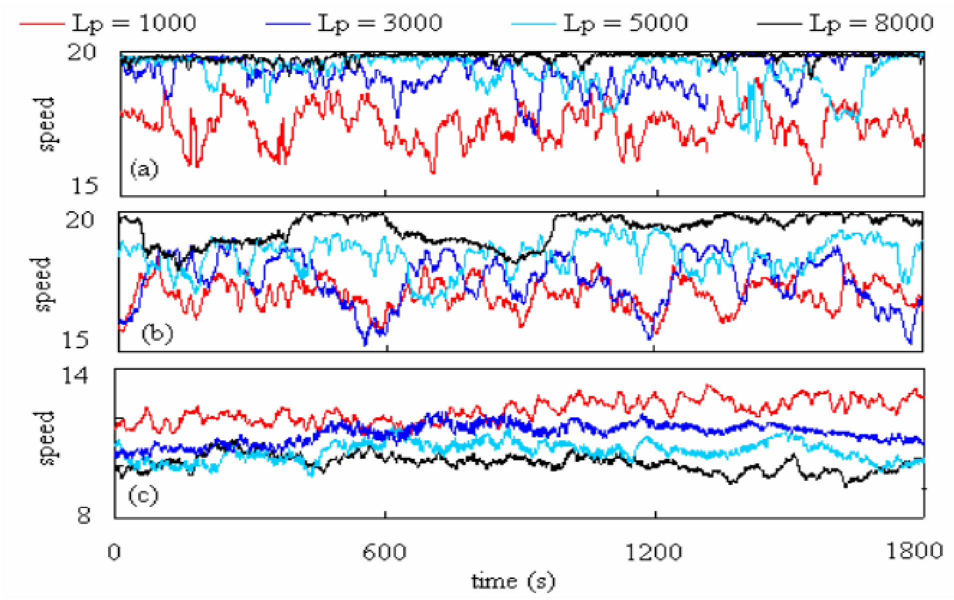

Fig. 9. (Color online) Mean speed against time after the relaxation of 20,000 time steps under different lengths of passing lanes. (a) density = 0.06; (b) density = 0.2; and (c) density = 0.4.

obvious increase of flux. Similarly, at high density, vehicles are in car-following flow. The speeds of fast vehicles depend strongly on the speeds of preceding vehicles. In this case, the velocities of fast vehicles in a passing lane normally do not reach their maximum, v^p_{max}. Therefore, flux decreases linearly with density and has little relation with v^p_{max}. However, at intermediate density, flux increases obviously when v^p_{max} increases. This is because more cars can drive on the passing lane with a higher speed than that in the normal condition (i.e., $v^p_{max} = 20$). This simulation suggests that allowing cars driving on a passing lane to travel at a higher speed can help to improve traffic further. For example, if a passing lane can be an unlimited speed zone, traffic flow may be improved significantly.

4.4. *Other considerations*

In simulations, we have found that passing lanes can help to merge jam regions, particularly, the longer the length of a passing lane, the narrower the jam region becomes. We implemented the simulation at density $\rho = 0.3$. The length of a passing lane is arbitrarily set to be 0 (without a passing lane), 600, 1,200 and 1,800 cells, respectively. Initially, vehicles (90% fast vehicles and 10% slow vehicles) are homogeneously distributed on a single-lane system. When the system reaches a stationary state, jam regions are stable (i.e., the length of each jam region almost remains fixed) and they can continue for considerable length of time for all four different lengths of passing lanes.

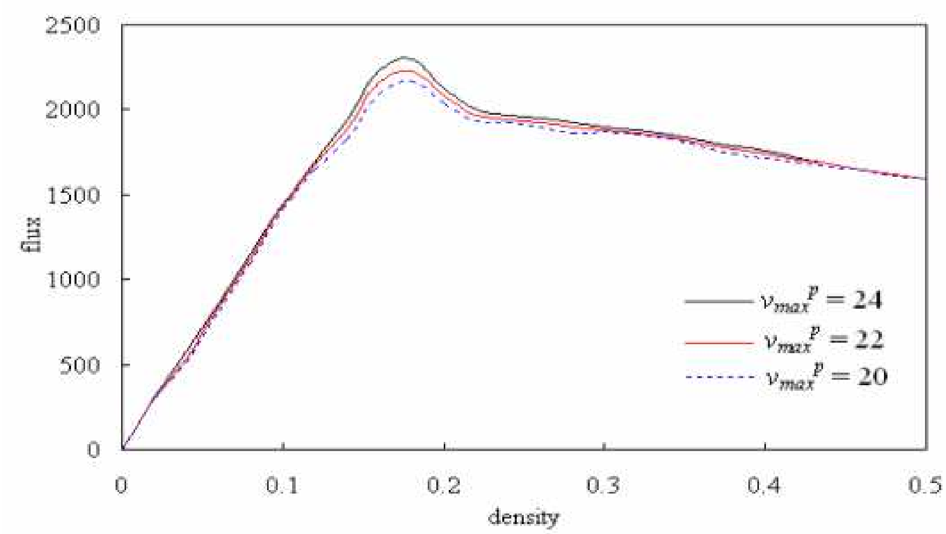

Fig. 10. (Color online) Fundamental diagram of heterogeneous flow (90% fast vehicles and 10% slow vehicles) with a passing lane. The length of the passing lane is assumed to be 1,000 cells. The maximum speeds of cars in the passing lane are assumed to be 24, 22, and 20 cells, respectively.

The space-time patterns are shown in Fig. 11, which are snapshots at the 3,600 time steps after 20,000 time steps have been discarded. Fig. 11(a) shows the evolutionary result of a single-lane system without a passing lane. It can be seen that 6 isolated jam regions exist. When we add a passing lane to a single-lane system, it is found that the isolated jam regions are merged into one or few wider regions, which is different from that without a passing lane. With increasing the length of a passing lane, the number of isolated jams decreases. This process is illustrated in Figs. 11(b)-(d). When traffic flow transits in a passing zone, the order of vehicles may change according to the lane-changing rules. As such, the positions of some vehicles are changed, which are analogous to impose a strong disturbance into the traffic flow. When a jam propagates to a passing zone, jams are formed in both the main and passing lanes. In this case, many vehicles will change to the passing lane as the passing lane has a higher priority to enter the single lane (lane D) than the main lane according to the yield rule.

5. Summary

In real traffic, passing lanes are used to provide overtaking opportunities for fast vehicles where two-lane highways are not available because of financial and/or geographical reasons and overtaking using an opposite-direction lane in a single-lane highway is often forbidden. Traffic flow going through a passing zone involves three processes: the diverging process, interacting and merging process. While a vehicle passes through a passing zone, it may be involved in the processes of lane selecting,

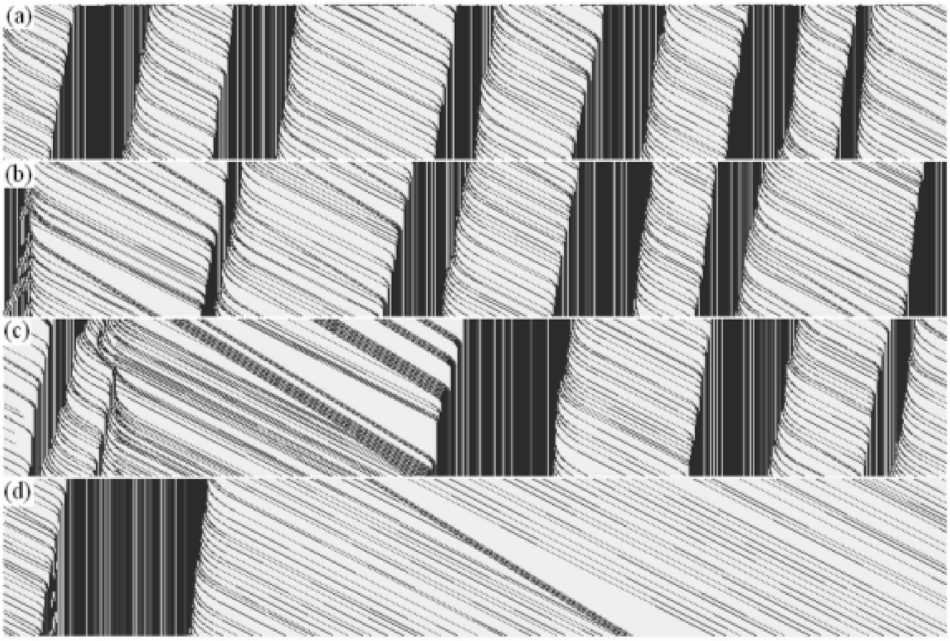

Fig. 11. Space-time spots at the density $\rho = 0.3$ with different length of a passing lane. (a) $L_p = 0$, (b) $L_p = 600$, (c) $L_p = 1{,}200$, (d) $L_p = 1{,}800$. Here different from Fig.2, vehicles drive from left to right and the time is increasing in downward direction.

lane changing (overtaking) and merging. When fast vehicles take the advantage of a passing lane to overtake their preceding vehicles, they change the vehicular configuration of traffic flow.

The effects of passing lanes on traffic are studied based on a Cellular-automaton (CA) model proposed by Lee et al. [37] under periodic boundary conditions. The focus is on using dual-time-headways (interaction headway and safe headway) to describe interaction between vehicles in the diverging and lane-changing processes. The dual-headway rule has a clear physical sense and can be further calibrated in the real world. Lane-changing rules are asymmetric since two types of vehicles (e.g., cars and trucks) are adopted and slow vehicles (trucks) are assumed to drive only on a main lane.

Numerical results have shown that the effects of passing lanes are related to the lengths of passing lanes, density, and the maximum passing speed. In the low- (or high-) density region, there are no obvious increases of flux due to very weak (or very strong) interactions between vehicles, regardless of the length of a passing lane. In the intermediate-density region, the flux of traffic flow has an obvious increase with the increase of the length of a passing lane. Additionally, if fast vehicles are allowed to drive their vehicles with a higher speed on a passing lane, traffic can be further improved, particularly at intermediate density. The space-time patterns are changed after vehicles go through a passing lane, compared to that before passing.

This is because vehicles have been rearranged in a passing zone.

We have found that passing lanes can merge jam regions, that is, the number of jam regions decreases when the passing lane is longer. We note that the decrease of the number of jam regions is at the cost of the increase of density on the passing lane. Thus, this decrease does not necessarily stand for the increase of the flux of the whole system.

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