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Synchronized flow and phase separations in single-lane mixed traffic flow

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Abstract

In this paper, we have studied synchronized flow and phase separations in mixed (heterogeneous) single-lane highway traffic. It is found that the flux–density (occupancy) curve of heterogeneous flow, as expected, lies in between two flux–density (occupancy) curves of homogeneous flow $R = 0$ (all vehicles are slow vehicles) and $R = 1$ (all vehicles are fast vehicles). However, unexpectedly, the velocity–density (occupancy) curve of heterogeneous flow does not. We also found that cross-correlation function (CCF) analysis shows that heterogeneous flow has almost the same strong coupling as homogeneous flow. In other words, when traffic is in free flow or jams, the value of CCF is approximate to be 1.0, while the value is about 0.1 in synchronized flow.

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1. Introduction

Vehicular traffic flow has attracted attention of statistical and nonlinear physical communities since the early 1990s. Various traffic models [1,2] have been presented. Among them, cellular automata (CA) have been widely used in modeling traffic flow on highways and in urban networks [3–12]. The reason for this is that CA are capable of modeling individual vehicle interactions and recapturing the macroscopic properties of traffic flow by aggregating the parameters obtained from the simulation.

The first stochastic traffic CA model for a single-lane highway was proposed by Nagel and Schreckenberg (NaSch model) [3], which is able to qualitatively reproduce some known traffic features (e.g., congestion, the flow–density relation and stop-and-go wave). Later, the NaSch model has been extended and different models have been developed. These models modified the acceleration rule [4,5], randomization rule [5–7] or deceleration rule [5,7–9] of the NaSch model. With these extensions, CA models are capable of simulating traffic flow more realistically. However, for the purpose of simplicity, only homogeneous traffic flow (the traffic flow contains only one type of vehicle) is simulated in most previous CA models.

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In the real world, traffic flow is normally a mix of several kinds of vehicles, e.g., cars, buses, vans and trucks. These vehicles with different dimensional and dynamic properties form heterogeneous traffic flow, which has been studied in recent years [13–18]. It is shown that mixed traffic flow can lead to platoon formations at low densities in single-lane traffic [13–18]. A platoon is a number of vehicles traveling together as a group, either voluntarily or involuntarily because of signal control, geometrics or other factors [19].

Treiber and Helbing [18] suggested a heterogeneous traffic flow to explain the phenomenon of the wide scattering (the empirical data points are distributed over a two-dimensional region) in congested states. The actual proportions of cars and trucks were used in their macroscopic traffic model. The simulation results show a high level of agreement with Dutch highway data. Other research indicated that the wide scattering might be caused by the anticipation effects of several vehicles ahead [20].

In addition, the mixed single-lane traffic has also been studied using the intelligent driving model (IDM) [21,22]. The similar fundamental diagram with 70% cars and 30% trucks is generated (see Fig. 17 in Ref. [21] and Fig. 19 in Ref. [22]). The effects of mixed vehicle lengths on traffic flow have also been studied in an asymmetric exclusion model [23]. Based on this model, the authors concluded that the maximal flux decreases when increasing the number of long vehicles. However, in the above-mentioned simulations, only a fixed proportion of cars and trucks is used. The effects of various car-truck proportions should be further studied, which is necessary for traffic and transportation management. Moreover, the formation of velocity under different car-truck proportions has not been reported so far.

Recently, Kerner et al. [24] investigated spatial-temporal structures of traffic flow and proposed a three-phase traffic flow theory, which postulates that there are three phases in traffic flow: free flow, synchronized flow and wide moving jams. Free flow and wide moving jams seem to be intuitively clear. Synchronized flow is mostly observed near on ramps or bottlenecks and characterized by a considerably high flux without any clear density-flux relation [24,25]. Thus, the study of synchronized flow has attracted much attention [5,7,10,20,25–28] in recent years. Many interesting phenomena and useful simulation results have been disclosed.

Kerner et al. [11] developed a CA model (KKW model) to simulate synchronized flow. Later, Kerner and Klenov [29] extended their KKW model [11] to two-lane heterogeneous traffic to investigate the effects of a variety of driver behavioral characteristics and vehicle parameters at on-ramps. The extended KKW model can reproduce the vehicle lane separation effect (fast vehicles use mostly the left (passing) lane, whereas other vehicles use the right lane) in free flow. In the KKW model (as well as its extensions), drivers are assumed to always attempt to keep the same speed with preceding vehicles within synchronization distances in order to reproduce synchronized flow [25]. In reality, it is difficult for a following driver to accurately predict the velocity of the preceding vehicle [30] and car-following is only one of many tasks that drivers perform simultaneously [31]. Thus, a certain range of velocity fluctuations of the following vehicle should be acceptable.

Ref. [32] investigated phase transition from free flow to synchronized flow in a multi-lane traffic. The authors introduced a variable h , called order parameter, to reflect degree of the internal interaction of vehicles along different lanes. In particular, a large value of the order parameter h characterizes synchronized flow, whereas free flow and the jam match its small values as weak mutual interactions (i.e., lane changing). In the cross-correlation function (CCF) between flux and density of traffic flow [5,33,34], synchronized flow is characterized by a small value of correlation coefficient $c_{k,J}$, free flow and the jams by a large value. Thus, the relation between order parameter h and CCF can be roughly represented by $c_{k,J} \approx 1 - h$.

In a recent publication [35], Huang adopted a simple CA model [36] to analyze the three different traffic phases in traffic flow on a homogeneous highway (i.e., without on ramps or bottlenecks). The simulation results have shown that these traffic phases can be reproduced on a homogeneous highway. That is, it is feasible to understand these traffic phases just from the point of view of vehicular interactions.

In this paper, we investigate synchronized flow and phase separations in single-lane heterogeneous traffic flow. Single-lane highways are very common in the countries like Australia, Ireland and New Zealand, because of financial and/or geographic reasons. These single-lane highways may equip short passing lanes for every 10 km or longer. Sometimes, a single-lane highway does not have a passing lane for up to 20 km. These stretches of single-lane highways may be just segments of an important and busy highway (e.g., several stretches of State Highway (SH) 1 and most part of SH 3 in the North Island of New Zealand). Because of

1 geographic reasons, passing using opposite-direction lanes is often forbidden. Especially, when a fast car
 2 follows a long, slow truck, passing is impossible on a busy highway. Thus, in this paper, passing using the
 3 opposite-direction lanes has not been considered. It can be seen that it is necessary to conduct the research in
 4 synchronized flow and phase separations of heterogeneous traffic flow on a *long single-lane* highway, which
 5 has not been reported in the literature, to the best of our knowledge.

6 To this end, we use the Jiang–Wu model [5] with the modification of acceleration rules. The Jiang–Wu
 7 model was derived from the braking-light (BL) model [7]. This paper is organized as follows. In Section 2, the
 8 single-lane Jiang–Wu model is described and the modification of acceleration rules is presented. In Section 3,
 9 simulation results and discussions of homogeneous and heterogeneous traffic systems are presented. The
 10 conclusion is given in Section 4.

11 2. The model

12 We first briefly describe the Jiang–Wu model (see [5] for a detailed explanation). Our modifications are then
 13 presented. The parallel update rules of the Jiang–Wu model are as follows:

14 (1) Determination of the randomization parameter p :

$$15 \quad p = p(v_n(t), b_{n+1}(t), t_h, t_s)$$

$$16 \quad p(v_n(t), b_{n+1}(t), t_h, t_s) = \begin{cases} p_b : & \text{if } b_{n+1} = 1 \text{ and } t_h < t_s, \\ p_0 : & \text{if } v_n = 0 \text{ and } t_{st,n} \geq t_c, \\ p_d : & \text{in all other case.} \end{cases}$$

17 (2) Acceleration:

18 If $(b_{n+1}(t) = 0 \text{ or } t_h \geq t_s)$ and $(v_n(t) > 0)$ then

$$19 \quad v_n(t+1) = \min(v_n(t) + 2, v_{max}) \quad (1)$$

20 else if $(v_n(t) = 0)$ then

$$21 \quad v_n(t+1) = \min(v_n(t) + 1, v_{max}) \quad (2)$$

22 else

$$23 \quad v_n(t+1) = v_n(t).$$

24 (3) Braking rule:

$$25 \quad v_n(t+1) = \min(d_n^{eff}, v_n(t+1)).$$

26 (4) Randomization and braking

27 If $(r < p)$ then

$$28 \quad v_n(t+1) = \max(v_n(t+1) - 1, 0).$$

29 (5) The determination of $b_n(t+1)$

30 If $(v_n(t+1) < v_n(t))$ then $b_n(t+1) = 1$.

31 If $(v_n(t+1) > v_n(t))$ then $b_n(t+1) = 0$.

32 If $(v_n(t+1) = v_n(t))$ then $b_n(t+1) = b_n(t)$.

33 (6) The determination of $t_{st,n}$

34 If $(v_n(t+1) = 0)$ then $t_{st,n} = t_{st,n} + 1$.

35 If $(v_n(t+1) > 0)$ then $t_{st,n} = 0$.

36 (7) The determination of $t_{f,n}$

37 If $(v_n(t+1) > v_c)$ then $t_{f,n} = t_{f,n} + 1$.

38 If $(v_n(t+1) < v_c)$ then $t_{f,n} = 0$.

39 (8) Vehicle motion

$$40 \quad x_n(t+1) = x_n(t) + v_n(t+1).$$

Here, x_n and v_n are the position and velocity of vehicle n (vehicle n follows vehicle $n + 1$); d_n is the gap between vehicle n and its preceding vehicle $n + 1$; b_n is the status of the brake light ($b_n(t) = 1$ or 0 means on or off); $t_h = d_n/v_n(t)$ is the time steps needed to reach the position of the preceding vehicle; $t_s = \min(v_n(t), h)$ is the velocity-dependent time steps to describe the interaction zone (where h is the braking range); $d_n^{eff} = d_n + \max(v_{anti} - gap_{safety}, 0)$ is the effective distance, where $v_{anti} = \min(d_{n+1}, v_{n+1})$ is the anticipative velocity of the preceding vehicle in the next time step; gap_{safety} is used to adjust the effectiveness of anticipation; $t_{st,n}$ denotes the time vehicle n stops; $t_{f,n}$ is the time that vehicle n remains in high speed; v_{max} denotes the maximum velocity; r is a random number uniformly distributed between 0 and 1.

The Jiang–Wu model can successfully describe light and heavy (characterized by large density and slow velocity with much fluctuation) synchronized flow in homogeneous traffic. In particular, the model can describe the so-called *comfortable braking process*. However, the model has not simulated the acceleration process properly. In the Jiang–Wu model, the acceleration process has been described as that a vehicle can speed up quickly as long as it is not stopped.

However, a high acceleration is normally found when a vehicle is in low speed, while a low acceleration is normally found when a vehicle is in high speed [38]. This is understandable. When a vehicle is in low speed (normally in a low gear), the force generated to move the car forward can be greater than the force at high speed. Thus, the acceleration can be greater.

In this paper, we thus modify the acceleration rules (Eqs. (1) and (2)) in the Jiang–Wu [5] model. The rest of the model is the same as the Jiang–Wu model. We modify the acceleration rules as follows:

If ($b_{n+1}(t) = 0$ or $t_h \geq t_s$) and ($v_n(t) > \frac{1}{2}v_{max}$) then

$$v_n(t+1) = \min(v_n(t) + 1, v_{max})$$

else if ($v_n(t) < \frac{1}{2}v_{max}$) then

$$v_n(t+1) = \min(v_n(t) + 2, v_{max})$$

$$\text{else } v_n(t+1) = v_n(t).$$

Basically, we assume the acceleration is high (i.e., equal to 2) when the speed has not reached half of the highest speed, while the acceleration is low (i.e., equal to 1) when the speed is higher than half the highest speed. This is the difference between our model and Jiang–Wu model. This assumption is consistent with our field observations and also agrees with [38]. In particular, as shown in Section 3.2, the cross-correlation analysis of our model agrees better with cross-correlation analysis of the empirical single-vehicle data in Ref. [33].

3. Simulation results and discussion

As our research is to investigate synchronized flow and phase separations in mixed traffic flow on *long single-lane* highways (described in Section 1), the periodic boundary condition is used in our computer simulations. The system size is set to $L = 10000$. One time step corresponds to 1 s. Each cell corresponds to a length of 1.5 m on a real road. Single-lane heterogeneous traffic flow is considered, which contains two kinds of vehicles: fast (e.g., cars) and slow (e.g., trucks) vehicles. The lengths of cars and trucks are set to 5 and 10 cells, respectively. R and R_t denote the proportions of cars and trucks to all vehicles (i.e., $R_t = 1 - R$). The maximum velocities of fast (cars) and slow (trucks) vehicles are assumed to be 20 cells (108 km/h) and 17 cells (91.8 km/h), respectively. This assumption is based on the recommended (or legal) speed limits for cars and trucks in New Zealand¹ and many other countries.

In this paper, we define a parameter of *occupancy* to represent the density of vehicles on a road. Normally, density is represented by the number of vehicles per kilometer in homogeneous traffic flow. However, the density cannot be accurately described in the traffic conditions of heterogeneous flow when the lengths of vehicles are different. Here, we define density as occupancy k , which is represented as a proportion of the number of occupied cells in the total number of cells.

¹The speed limit of cars is 100 km/h in New Zealand. However, police will not issue a ticket unless the speed is above 111 km/h. Thus, most drivers of cars tend to keep their speeds just below 110 km/h.

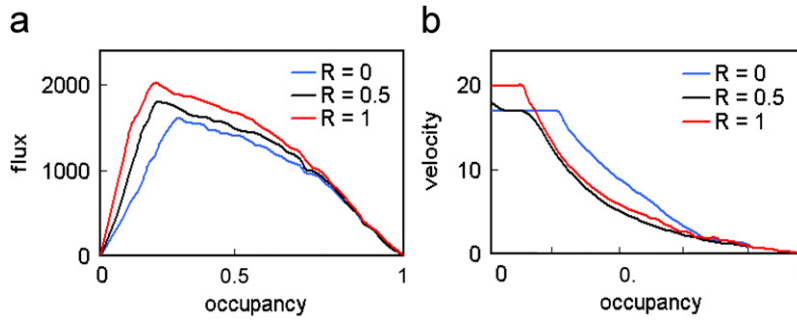


Fig. 1. (a) The fundamental diagram under different R and (b) the occupancy–velocity plot under different R . R is the proportion of fast vehicles to all vehicles.

In the simulations, the values of other parameters in our model are set to $t_c = 10$, $t_{c1} = 30$, $v_c = 18$ for cars, $v_c = 15$ for trucks, $p_d = 0.1$, $p_b = 0.94$, $p_0 = 0.5$, $h = 6$. If the leading vehicle is a truck, the following truck driver, as well as car driver, cannot see through other vehicles in front of them. We thus assume that $gap_{safety} = 10$. In other cases, $gap_{safety} = 7$.² Each simulation is carried out for 360 000 time steps (i.e., 100 h) after the relaxation of 20 000 time steps. In order to remove the influence of random initial conditions on traffic flow, we have repeatedly implemented each simulation 20 times. We use an averaged flux and velocity to measure the fundamental diagram. In the following simulations, when we consider the occupancy changes, the occupancy increases with 2% increments.

3.1. Fundamental diagrams

We examine homogeneous traffic flow first. Fig. 1(a) shows the diagram of flux versus occupancy under different R (R is the proportion of fast vehicles to all vehicles). Based on our simulation, we found that flux–occupancy curves are almost the same when $0 < R < 1$ as well as velocity–occupancy curves. Thus, we only show flux–occupancy and velocity–occupancy curves for $R = 0.5$. In Fig. 1(a), we can see that the flux–occupancy curve of heterogeneous flow lies in between two flux–occupancy curves of homogeneous flow $R = 0$ (all are slow vehicles) and $R = 1$ (all are fast vehicles). When $R = 0$, there are only trucks on the road. The maximum velocity of vehicles is 17; The maximum flux is about 1622 when the occupancy is 0.30. When $0 < R < 1$, the flow is a mixture of cars and trucks. The flux of heterogeneous flow is larger than that of $R = 0$ due to the shorter dimension of cars. However, with the increase of occupancy (> 0.7), the flux of heterogeneous flow is nearly the same as homogeneous flow. When $R = 1$, there are only cars on the road. In this case, the maximum average velocity of the vehicles can reach up to 20; the maximum average flux is about 2034 vehicles per hour when the occupancy is 0.16. These results are consistent with Refs. [16,17,23]. The results show that lower maximum velocity (17 cells) and longer dimension (10 cells) of vehicles can decrease the flux.

Fig. 1(b) shows the diagram of velocity versus occupancy for different R . It can be seen that the velocity–occupancy curve of heterogeneous flow does not lie in between two velocity–occupancy curves of homogeneous flow $R = 0$ (all are slow vehicles) and $R = 1$ (all are fast vehicles). This is different from the flux–occupancy curve in Fig. 1(a). In Fig. 1(b), when occupancy < 0.16 , traffic is free flow when $R = 1$ (all cars), thus the velocity is higher than that when $R < 1$. When $R < 1$, the velocity is nearly equal to the highest speed of slow vehicles. In other words, increasing or decreasing the number of trucks within a certain range does not influence traffic flow qualitatively. When the occupancy is lower (< 0.16), we can see that slow vehicles determine the speed of heterogeneous traffic flow. This is consistent with the empirical observations described in Ref. [17] and qualitative analysis discussed in Refs. [16,23].

²A real world driver will use a braking rate based on the premonitory comprehensive information of several cars in front, rather than only on the information of one vehicle in front. A corollary to this is that a driver of a small vehicle would be more cautious and allow more space if driving behind a huge size vehicle (e.g. a truck). Typically, the small following vehicle is unable to get any information about other vehicles in front in such situation.

1 When occupancy is larger than 0.16, the velocity with $R = 0$ (all trucks) can keep a higher value than that
 2 with $R > 0$. With the same occupancy, the average spaces between vehicles will decrease as R increases. In
 3 other words, when $R = 0$, there is more space between slow vehicles compared to mixed vehicles ($0 < R < 1$) or
 4 fast vehicles only ($R = 1$). Thus, with the same occupancy, there are three cases: (i) $R = 0$, larger space can
 5 make slow vehicles drive with higher velocity (see Fig. 1(b)). (ii) $R = 1$, the average space between vehicles is
 6 the smallest compared to mixed and slow vehicles. As fast vehicles have the maximum desired speed than
 7 mixed and slow vehicles, stronger interactions between fast vehicles and with less space between vehicles lead
 8 to decreased velocity of fast vehicles. (iii) $0 < R < 1$, when a car is following a truck, a larger safety distance
 9 ($gap_{safety} = 10$) is needed. Thus, the speed of the following is the same or less than the preceding. The velocity
 10 of vehicles is actually lower than both the velocity with $R = 0$ or 1.

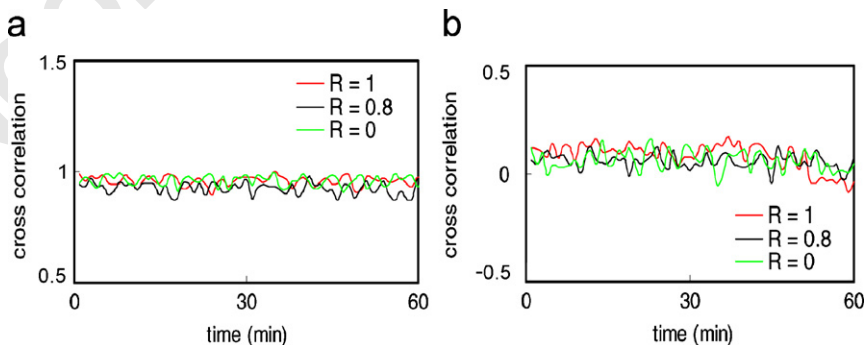
11 3.2. Cross-correlation analysis

12 The fundamental diagram can describe the general trend of traffic states. However, it is not sufficient to
 13 accurately identify the different types of traffic states (i.e., free flow, synchronized flow and wide moving jams)
 14 as indicated in Ref. [37]. A more clear description of traffic states can be obtained by the cross-correlation
 15 analysis suggested in Refs. [5,33,34].

16 We plotted the CCF between occupancy and flux averaging 20 times in order to remove the influence of
 17 random initial conditions. Fig. 2(a) shows the cross correlation between flux and occupancy at low occupancy
 18 ($k = 0.06$). It can be seen that mixed flow ($R = 0.8$) has almost the same correlation in free-flow state as
 19 homogeneous ($R = 0$ and 1), i.e., $c_{k,J} \approx 1$. These cross-correlation results are consistent with cross-correlation
 20 analysis of the empirical single-vehicle data in Ref. [33]. However, the cross-correlation factor of homogeneous
 21 traffic is slightly more stable than that of heterogeneous traffic in Fig. 2(a). This can be explained as platoons
 22 occur even when the mixed flow is in low occupancy.

23 Fig. 2(b) shows the cross correlation between flux and occupancy when $k = 0.5$. We find that the cross-
 24 correlation fluctuates around 0.1, i.e., $c_{k,J} \approx 0.1$ for both heterogeneous and homogeneous traffic. The cross-
 25 correlation factor of mixed traffic is slightly better than homogeneous traffic in Fig. 2(a). This is because the
 26 interactions between vehicles in mixed traffic flow is less intensive when the vehicles are with lower velocity (see
 27 Fig. 1 (b)) and at high occupancy ($k = 0.5$).

28 In particular, for homogeneous traffic, we note that the cross-correlation factor of our model is slightly
 29 higher than in that of the Jiang–Wu model [5]. This implies that the vehicles in our model interact stronger
 30 than those in the Jiang–Wu model. This is because we have considered a higher acceleration in low speed. Our
 31 results thus agree better with cross-correlation analysis of the empirical single-vehicle data in Ref. [33].



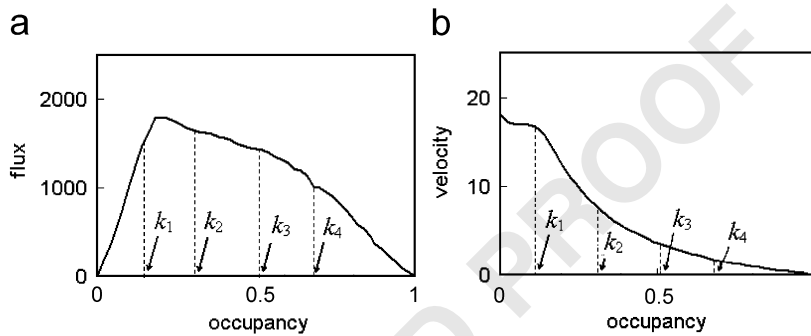
32 Fig. 2. (Color online) Cross-correlation functions between flux and occupancy plotting at different R : (a) occupancy $k = 0.06$ and (b)
 33 occupancy $k = 0.50$.

1 3.3. Phase transitions

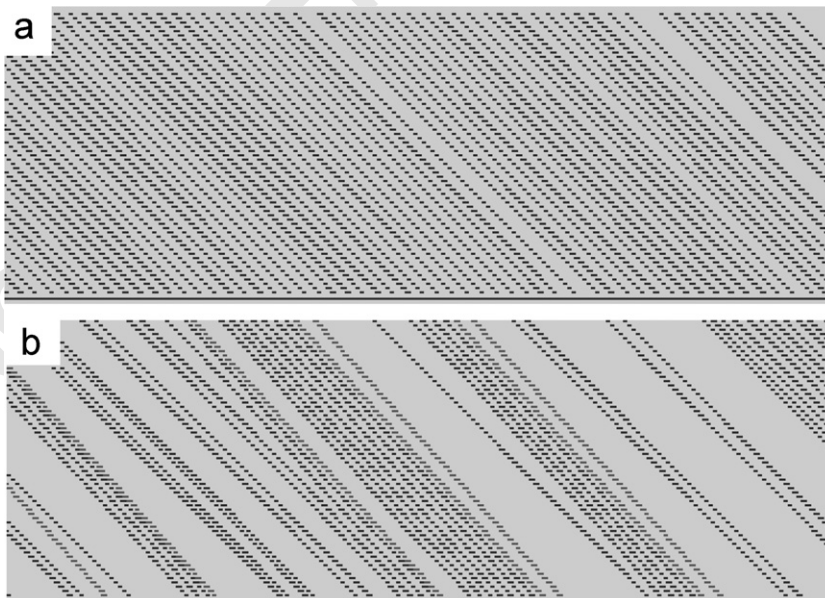
3 The fundamental diagram of our model and the phase transition from free flow to synchronized flow in
 4 homogeneous traffic are similar to that in Ref. [5]. In this section, we simulate synchronized flow and phase
 5 separations in heterogeneous traffic where the system contains 80% cars and 20% trucks, i.e., $R = 0.8$. Five
 6 occupancy ranges are divided in Fig. 3. Occupancy k_1 corresponds to the maximum occupancy where the
 7 average velocity is nearly 17. Occupancy k_2 corresponds to the maximum occupancy where the traffic reaches
 8 the maximum flux. Densities k_3 and k_4 correspond to an occupancy range where a phase separation occurs.

9 When occupancy is small ($k < k_1$), space–time patterns of heterogeneous flow (see Fig. 4(b)) are quite
 10 different from the homogeneous flow (see Fig. 4(a)). In the homogeneous traffic flow, the flow is in a free-flow
 11 phase. In the heterogeneous traffic flow, the flow has platoons due to the blockage of slow vehicles (Fig. 4 (b)).
 12 It can be seen that synchronized flow exists inside the platoons and free flow exists between platoons in Fig. 4
 13 (b).

14 When $k_3 < k < k_4$, a phase separation between synchronized flow and wide moving jams occurs.
 15 Synchronized flow can be maintained for a period of time. The longest period of time found in our
 16 simulation is $\Delta t = 5030$. After flow in a synchronized flow phase, the system transits into wide moving jams
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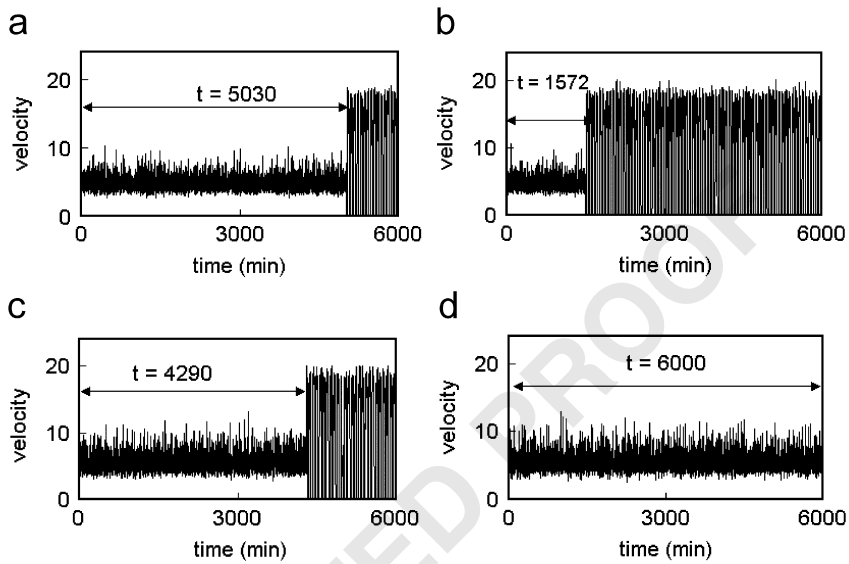


29 Fig. 3. Fundamental diagram and corresponding velocity–occupancy diagram of our model in a heterogeneous system at $R = 0.8$.

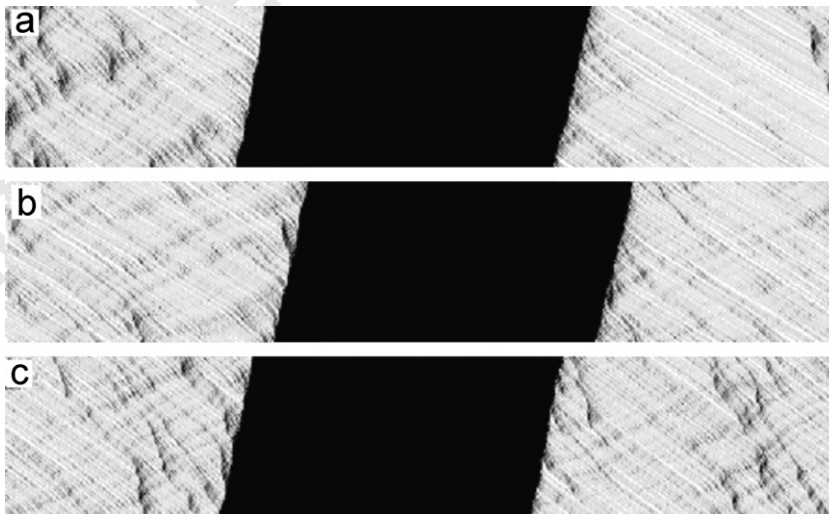


51 Fig. 4. Space–time plots at occupancy $k = 0.06$: (a) 100% cars and (b) 80% cars and 20% trucks.

1 due to random disturbance (see Fig. 5(a)). The phenomenon has been found both in homogeneous and
 2 heterogeneous flow. We also find that the periods of synchronized flow maintained are different depending on
 3 the initial conditions of the system in heterogeneous flow. For instance, the initial scattering of the vehicles in
 4 the experiments shown in Fig. 5(a) and (b) was different (because the random numbers generated in each
 5 experiment are different), the periods of synchronized flow maintained are thus different. The simulation
 6 results also show that the average value of Δt decreases with the increase of occupancy in this occupancy range
 7 ($k_3 < k < k_4$). According to our simulation, we also find that the period of time Δt in homogeneous flow (i.e.,
 8 pure cars or trucks) is stable and not very sensitive to random initial conditions, which is different from
 9 heterogeneous flow mentioned above. Moreover, the period of time Δt varies with type of vehicles. In Fig. 5(c),
 10 when $R = 1$ (all cars), the flow evolves into jams from synchronized flow after 4290 min. Whereas $R = 0$ (all
 11 trucks), the flow keeps synchronized flow throughout our simulation (100 h).



31 Fig. 5. One-minute average velocity of traffic flow obtained at a virtual detector. The occupancy is $k = 0.5$: (a) and (b) the proportion of
 32 cars $R = 0.8$; (c) the proportion of cars $R = 1$; and (d) the proportion of cars $R = 0$.



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Fig. 6. Space-time plots under different proportion of vehicles. The occupancy is $k = 0.5$: (a) $R = 1$; (b) $R = 0.8$; and (c) $R = 0$.

1 We also examine the outflow from jams. To this end, we set the initial condition as a mega jam. The outflow
from jams in our simulation is coexistence of free flow and light synchronized flow in this occupancy range
3 ($k_3 < k < k_4$) for homogeneous and heterogeneous flow (see Fig. 6) when the initial condition starts from a
mega jam. This agrees with the Jiang–Wu model [5] and the empirical observations in Ref. [24].

7 4. Conclusions

9 In this paper, we have modified the acceleration rules in the Jiang–Wu model [5] to correct a defect in the
model. The cross-correlation analysis has shown that our model agrees better with the cross-correlation
analysis of the empirical single-vehicle data in Ref. [33].

11 We have applied our model to investigate synchronized flow and phase separations in single-lane mixed
traffic flow. Our simulation results have shown the following two important features: (i) the flux–occupancy
13 curve of heterogeneous flow, as expected, lies in between two flux–occupancy curves of homogeneous flow
 $R = 0$ (all are slow vehicles) and $R = 1$ (all are fast vehicles). However, unexpectedly, the velocity–occupancy
15 curve of heterogeneous flow does not. (ii) cross-correlation function (CCF) analysis shows that heterogeneous
flow has almost the same strong coupling as homogeneous flow. In other words, when traffic is in free flow
17 (e.g., $k = 0.06$), the value of CCF is about 1, while it is about 0.1 in synchronized flow (e.g., $k = 0.5$).

19 Our simulation results have also confirmed some previous perceptions and findings: (i) when the occupancy
is low (< 0.16) slow vehicles dominate traffic flow even when their number is small, which is consistent with the
empirical observations and qualitative analysis described in Refs. [16,17,23]; (ii) there is synchronized flow
21 inside platoons and free flow outside platoons in heterogeneous traffic flow at low occupancy; (iii) the period
of time Δt to evolve into jams from synchronized flow varies with random initial conditions in heterogeneous
23 flow, however, it is relatively stable in homogeneous flow compared to heterogeneous flow; and (iv) free flow
and light synchronized flow coexist in the outflow from jams in both homogeneous and heterogeneous flow
25 when starting from a mega jam. This agrees with the simulation in the Jiang–Wu model [5] and empirical
observations described in Ref. [24].

27 In future work, we will extend the current work to investigate mixed traffic flow on single-lane highways
with passing lanes. Passing lanes can alleviate the influence of slow vehicles in a certain degree, which have
29 been widely used in many countries.

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