Abstract - This paper discusses the sensitivity of network flows to uncertain link state information for various routing protocols. We show that the choice of probability distribution for the link metrics for a given network can have markedly different effects on the probabilities of path selection. Exact results are obtained for these probabilities but their computation is NP-hard. We provide simulation results for three networks to illustrate the sensitivity of shortest paths to different link metric distributions. We provide results for mean path costs and the k-shortest path algorithm as a comparison.

1. INTRODUCTION

Commonly used Internet routing protocols include the Routing Information Protocol (RIP) and Open Shortest Path First (OSPF). Currently, OSPF is one of the preferred interior routing protocols. Detailed information on the routing of Internet traffic is available in Huitema [11]. Recent years have seen major research activities in developing Quality of Service (QoS) networks. Wang [12] gives a detailed description of QoS and the main architectures proposed for achieving QoS requirements.

QoS routing enables a network to identify paths for any new flow with adequate resources to meet the flow’s requirements, typically bandwidth or end-to-end delay guarantees. It is expensive to frequently update state information specifying resource availability and hence these updates may only take place infrequently or imprecisely by aggregating network states. Guérin and Orda [1] stress that the task of finding a path capable of accommodating a new request is very much dependent on the accuracy of information on network resources. In the case of available bandwidth a number of modifications such as widest-shortest paths have been proposed. See [1], [2], [4], [5], [9] for details. Kowalik and Collier [10] have recently proposed a new algorithm for generating link state updates.

These uncertainties can affect routing decisions when they depend on uncertain link state information [1], [2]. Guérin, Orda, Apostopoulos and other researchers have been very active in proposing new ideas for handling edge state uncertainty [3], [4], [5]. Information on link utilisation, such as that obtained via packet sniffers, may be used to estimate suitable proportions for splitting traffic flows.

The choice of a shortest path from an origin to a destination clearly depends on the values of the edge costs. In an OSPF domain, edge weights can take several different values, depending on the type of service. The actual values are often subject to uncertainty, due to the ageing of information held in the router tables. Thus, we can view link utilisation as a random variable. This has a flow-on effect regarding queueing delays experienced by packets arriving at a router. Hence, we can also treat edge delays as random variables. Delay variation is also subject to random variation in the packet arrival process.

Henceforth, we shall restrict our attention to OSPF domains. This paper gives examples to show that the choice of edge cost distribution strongly influences the probability of a given path being selected as the shortest path. Exact formulae exist but are too complex for practical use.

We have developed a program to simulate the stochastic variation of link costs, using a range of possible distributions for the link costs. The program is used to simulate the choice of a shortest path, according to the random path cost. Over a large number of trials, statistics are gathered on the frequency of path selections.

In an interesting parallel to our work, Hutson and Shier [6] considered the possibilities of a probability distribution for the edge weights in a Minimum Spanning Tree (MST), where the edge weights are independent discrete random variables. Their objective was to analyse the distribution of possible MSTs. In contrast, our focus is mainly on a single traffic demand and the selection of one path from the possible shortest paths, depending on the edge weights.

Several authors [6], [7], [8] considered network optimization problems where the edge weights are independent random variables, which need not be identically distributed. It is shown in [7] that the exact calculation of the distribution of the shortest path tree weights is NP-hard. In detail, let s be a source node and let t be any other node. Suppose that the edge weights are described by discrete probability distributions. An (s,t) path is the shortest path from s to t for a realisation of these edge weights. The following problems are shown in [7] to be NP-hard. These problems are listed below:

\[ F(x) = \Pr(\exists(s,t) \text{ path of length } \leq r) \] for some fixed r.

\[ c(e) = \Pr(\text{edge } e \text{ is on a shortest path}) \]

\[ c(P_0) = \Pr(\text{Path } P_0 \text{ is shortest}) \]
2. ANALYTICAL RESULTS

Let us consider a network with \( n \) nodes and \( m \) links. We consider the task of choosing a route from a source node \( s \) to a destination node \( d \). Let \( r \) be the number of paths allowed to be chosen as a route from \( s \) to \( d \). Label these \( r \) paths \( P_1, \ldots, P_r \). Denote the cost of path \( P_i \) by \( C_i = C(P_i) \). Without loss of generality, we can rank these paths in order of their mean path costs \( \mu_i = E(C_i) \), so that \( \mu_1 \leq \cdots \leq \mu_r \).

Let path \( P_i \) have link cost distribution \( F_i(t) \) with associated density function \( f_i(t) = F_i'(t) \) for \( i = 1, \ldots, r \). These notations will enable us to write down the probability of selecting path \( P_i \) for \( i = 1, \ldots, r \).

The probabilities for the maxima and minima of \( n \) random variables \( X_1, \ldots, X_n \), corresponding to the link metrics, assumed to be mutually independent, are given by

\[
\Pr(\max(X_1, \ldots, X_n) \leq t) = F_1(t) \cdots F_n(t)
\]

\[
\Pr(\min(X_1, \ldots, X_n) \leq t) = 1 - (1 - F_1(t)) \cdots (1 - F_n(t))
\]

Note that different paths from \( s \) to \( d \) need not be independent since they can have one or more links in common. Nevertheless, we can find the path selection probabilities. Next, we obtain the probability \( p_i \) that the maximum or minimum of \( n \) mutually independent RVs equals a given RV \( X_j \). This will yield the probability of selecting the \( i \)th path. Let \( Y = \max(X_1, \ldots, X_n) \) and \( Z = \min(X_1, \ldots, X_n) \). Let \( \Re \) denote the set of real numbers. The event \( \{Y = X_i\} \) can be written in the form

\[
\{Y = X_i\} = \bigcup_{j \in \Re} \{ \omega : X_j(\omega) = t & X_j(\omega) \leq t, j \neq i \}
\]

and hence its probability is

\[
\Pr(Y = X_i) = \int_{\Re} \Pr(\omega : X_j(\omega) = t & X_j(\omega) \leq t, j \neq i) \, dt
\]

\[
= \int_{\Re} F_i(t) \cdots F_{i-1}(t) f_i(t) F_{i+1}(t) \cdots F_n(t) \, dt
\]

Similarly, the event \( \{Z = X_i\} \) can be written in the form

\[
\{Z = X_i\} = \bigcup_{j \in \Re} \{ \omega : X_j(\omega) = t & X_j(\omega) > t, j \neq i \}
\]

and hence its probability \( p_i \) is

\[
\Pr(Z = X_i) = \int_{\Re} \Pr(X_j(\omega) = t & X_j(\omega) > t, j \neq i) \, dt
\]

\[
= \int_{\Re} (1 - F_i(t)) \cdots (1 - F_{i-1}(t)) f_i(t) \, dt
\]

\[
(1 - F_{i+1}(t)) \cdots (1 - F_n(t)) \, dt
\]

If the path \( P_i \) has the distribution function \( F_i(t) \) the probability that it will be chosen is given by \( p_i \) as expressed above. This result is not suitable for explicit evaluation as the integrand is rather complex. Indeed, it can be shown that the problem is NP-hard.

Link metrics are assumed in this work to be additive, so the link cost metrics are of the form \( C_i = C_{i,1} + \cdots + C_{i,k_i} \), where \( C_{i,j} \) is the cost of the \( j \)th link on the \( i \)th path. The distribution for these sums can be found using convolutions of random variables. See Moran [14] or Feller [15] for details.

However, these theoretical results are too complex for practical use. Routers need to make their decisions rapidly and “on the fly”. Hence it is desirable to develop simple approaches for estimating the selection probabilities for different paths. One possible approach could be to set up a regression model for the proportion \( p_i \) of traffic from a source \( s \) to a destination \( d \) that is routed along path \( P_i \).

It is difficult to rigorously derive a regression model for the theory given above. Using standard results on extreme value theory (see [16]) for a normal distribution, we can suggest heuristic models based on the following pattern:

\[
\log p_i = A + B \log r + C \frac{\mu_i}{\sigma} \sqrt{\log r} + D \frac{\sigma \epsilon_i}{\sigma} \sqrt{\log r} + E \frac{\overline{p}_i}{\sigma} \log r + \epsilon_i,
\]

where \( E \epsilon_i = 0 \), \( E \epsilon_i^2 < \infty \) and \( A, B, C, D \) and \( E \) are slope parameters to be determined and \( \epsilon_i \) is a random error. Here we set \( \overline{p}_i = \frac{1}{r} \sum_{i=1}^{r} \mu_i \) and \( \sigma^2 = \frac{1}{r} \sum_{i=1}^{r} \sigma_i^2 \). A simpler model with fewer terms could be used as it is easier to interpret. This type of model has the virtue of being relatively insensitive to the value (\( r \)) of the number of possible paths. The factor \( \sqrt{\log r} \) is a consequence of extreme value theory for normal distributions [16]. However, the model will only be useful if the coefficients are relatively insensitive to different link weight distributions and different topologies.
3. SOME EXPERIMENTAL RESULTS

In this section, we describe some examples that show the effects of link weight distributions on the path selection of a shortest path algorithm. Our study assumes a generic undirected network where the links have randomly assigned weights from selected probability distributions. For our experiment, we used scaled versions of the following distributions: Uniform(0,1) - UD, symmetric Beta(2, 2) - BSD, right skewed Beta(1.5, 5)–BRS and left skewed Beta(5, 1.5)– BLS. These are representative examples of realistic link metric distributions. It was assumed that all link weight distributions in one network are of the same type.

Figure 1. ATM network

A simulator was written in C++ for this purpose. This program accepts as input a network topology together with its link costs. The user also selects an Origin-Destination (OD) pair. A set of link weights is generated randomly according to the selected probability distribution and Dijkstra’s algorithm is used to find a shortest path from origin to destination. This experiment was repeated 100,000 times.

Path selection frequencies were tallied for all paths that turned up in these experiments. We also give standard errors (SEs) in parentheses below the estimate. To find a 95% confidence interval, we simply multiply the SEs by ±1.96 and use them to form the lower and upper bounds. For example, the estimated frequency of path 1-6 in FM under the UD scenario is 52.61% with an SE of 0.16%. Hence, a 95% confidence interval is (52.30%, 52.92%).

We considered three different network topologies: A six node fully meshed network (FM), a 15 node 28 links network shown in Fig. 1 (ATM), and a 28 node 45 links network (USA). The fully meshed example is typical of a logical network overlaying an optical network. The other two networks are fully described in [13] and represent typical topologies. Note that the lower limit for link weights was always taken as zero. Hence, only the upper limits for the link weight are shown in Fig. 1. Thus the number ‘185’ on link 1-2 means that its link weight randomly varies between 0 and 185.

We compared our results with the shortest paths obtained by using the mean (expected) path costs. We also compared these results with a list of paths obtained by using a k-shortest path algorithm. A large number of paths can potentially be selected as the shortest path, according to the actual link weights, so only some of the more common choices are presented.

Figure 2. Results for ATM network.

The results for the FM network are depicted in Table 1. The OD pair was 1 – 6. The lower limit on the link weight was set to 0 and the upper limit was set to 10 for all links. The first column depicts the most common paths and the subsequent column show the relative frequency with which they appeared for the different distributions. In this case, the one hop route from ‘1’ to ‘6’ is the most frequent choice, but the selection frequency depends on the choice of link weight distribution. For the FM network, the mean link costs are all the same so they are equivalent to the hop count metric. The one-hop paths are always the shortest paths in this case.

A route with two hops can only be selected if the sum of the two link costs on the selected path is less than the cost of the direct one-hop route. Thus we consider the event \( X + Y < Z \) where \( X, Y, Z \) are all random variables between 0 and 1. When \( X, Y, Z \) are all uniformly distributed, we find that this

<table>
<thead>
<tr>
<th>Paths</th>
<th>UD</th>
<th>BSD</th>
<th>BRS</th>
<th>BLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-6</td>
<td>52.61% (0.16%)</td>
<td>68.9% (0.15%)</td>
<td>54.6% (0.16%)</td>
<td>98.6% (0.04%)</td>
</tr>
<tr>
<td>1-2-6</td>
<td>8.8% (0.09%)</td>
<td>7.0% (0.08%)</td>
<td>8.9% (0.10%)</td>
<td>0.3% (0.02%)</td>
</tr>
<tr>
<td>1-3-6</td>
<td>8.7% (0.09%)</td>
<td>6.8% (0.08%)</td>
<td>9.0% (0.10%)</td>
<td>0.3% (0.02%)</td>
</tr>
<tr>
<td>1-4-6</td>
<td>8.8% (0.09%)</td>
<td>7.0% (0.08%)</td>
<td>8.8% (0.10%)</td>
<td>0.3% (0.02%)</td>
</tr>
<tr>
<td>1-5-6</td>
<td>8.8% (0.09%)</td>
<td>6.9% (0.08%)</td>
<td>9.0% (0.10%)</td>
<td>0.3% (0.02%)</td>
</tr>
<tr>
<td>Other paths</td>
<td>12.3% (0.1%)</td>
<td>3.4% (0.06%)</td>
<td>9.7% (0.09%)</td>
<td>0.2% (0.02%)</td>
</tr>
</tbody>
</table>
event has probability $1/12$. This is in accord with the second column of Table 1 after an obvious scaling. The third and fourth columns are similar. The BLS scenario yields link costs that are close to the upper limit. So the case where the cost of two hops is below that of a one hop path is unlikely.

<table>
<thead>
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<th>BSD</th>
<th>BRS</th>
<th>BLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5-11-12-15</td>
<td>45.63% (0.16%)</td>
<td>16.50% (0.12%)</td>
<td>11.21% (0.10%)</td>
<td>40.95% (0.16%)</td>
</tr>
<tr>
<td>1-5-11-14-15</td>
<td>15.39% (0.11%)</td>
<td>15.48% (0.11%)</td>
<td>11.18% (0.10%)</td>
<td>27.79% (0.14%)</td>
</tr>
<tr>
<td>1-5-8-14-15</td>
<td>1.67% (0.04%)</td>
<td>10.60% (0.10%)</td>
<td>9.17% (0.09%)</td>
<td>13.82% (0.11%)</td>
</tr>
<tr>
<td>1-3-6-7-12-15</td>
<td>37.31% (0.15%)</td>
<td>4.83% (0.07%)</td>
<td>3.96% (0.06%)</td>
<td>3.28% (0.06%)</td>
</tr>
<tr>
<td>Other paths</td>
<td>0%</td>
<td>52.89% (0.16%)</td>
<td>64.48% (0.15%)</td>
<td>14.16% (0.11%)</td>
</tr>
</tbody>
</table>

The results for the ATM network are shown in Table 2, where the origin is ‘1’ and the destination is ‘15’. This case shows more variation in path selection frequencies. The path with the lowest mean path cost is 1-5-11-12-15. This path is also the most frequent choice when the weight distribution is uniform. This is also the case for BLS. For BRS and BSD, no single path is frequently selected. These results are shown in Fig. 2.

<table>
<thead>
<tr>
<th>Paths</th>
<th>UD</th>
<th>BSD</th>
<th>BRS</th>
<th>BLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5-9-10-11-25-26-28</td>
<td>68.97% (0.15%)</td>
<td>34.11% (0.15%)</td>
<td>19.43% (0.13%)</td>
<td>81.61% (0.16%)</td>
</tr>
<tr>
<td>1-2-3-4-13-14-21-23-27-28</td>
<td>31.03% (0.15%)</td>
<td>15.34% (0.11%)</td>
<td>10.18% (0.10%)</td>
<td>12.68% (0.11%)</td>
</tr>
<tr>
<td>1-2-7-10-11-25-26-28</td>
<td>0%</td>
<td>4.64% (0.07%)</td>
<td>9.00% (0.09%)</td>
<td>0.87% (0.10%)</td>
</tr>
<tr>
<td>1-5-9-10-11-22-26-28</td>
<td>0%</td>
<td>5.70% (0.07%)</td>
<td>8.80% (0.09%)</td>
<td>1.34% (0.04%)</td>
</tr>
<tr>
<td>1-2-3-4-8-12-11-25-26-28</td>
<td>0%</td>
<td>5.80% (0.07%)</td>
<td>4.80% (0.09%)</td>
<td>1.68% (0.04%)</td>
</tr>
<tr>
<td>Other paths</td>
<td>0%</td>
<td>34.41% (0.02%)</td>
<td>47.79% (0.16%)</td>
<td>1.82% (0.08%)</td>
</tr>
</tbody>
</table>

For the four link weight distributions, we find that the four listed paths from top to bottom are ranked 1 (smallest), 2, 3 and 8 respectively with regard to mean path cost. Also, their hop counts are 4, 4, 4 and 5 respectively. Thus paths with small mean path costs or small hop counts tend to be selected more frequently than other paths.

The results for the USA network are shown in Table 3 where the origin is ‘1’ and the destination is ‘28’. In this case, the ranks of the five listed paths from top to bottom vary according to the link weight distribution. The first listed path is ranked 1 (smallest) in order of mean path cost under all scenarios. The second listed path ranks 2 for UD and BSD and 3 for BRS and BLS. The remaining three paths have hop count 7 while the second and fifth have hop count 9. The likelihood of selection is generally largest for paths with small mean cost. However, mean path cost is only an approximate predictor of its chance of selection as Table 3 shows.

The results show that different distributions on the link-metrics can lead to different frequencies of routes actually selected. However, paths with lower expected costs are more likely to be selected than other paths. Some typical results for proportion of shortest path selection versus mean cost are shown in Fig. 3 and 4. There is clearly a linear trend between mean path cost and the logarithm of the probability of path selection for both the ATM and USA networks, where paths
of lowest mean path cost have the highest probability of selection.

The graphs for probability of path selection versus mean path cost takes a similar form for the ATM and USA networks. Only two of them are illustrated as the rest are found to be similar. A regression model could be fitted, but the regression coefficients depend on both the topology and the link metric distribution. Hence a regression model is not particularly useful for traffic engineering purposes.

4. CONCLUSIONS AND FURTHER WORK

We have demonstrated that the link metric distribution can profoundly affect the selection of shortest paths in a network. This effect has been illustrated for several topologies. Moreover, the frequency of path selection also depends on the assumed probability distribution. We have found that the logarithm of the probability of path selection depends approximately linearly on the mean path cost. These observations have significant consequences for traffic management if we use stochastically varying metrics such as measured delay or the inverse of available link capacity as the link metric.

Further work is being carried out on possible methods of routing traffic demands on two or more alternate paths where each alternate path is selected with a prescribed probability. The simplest way would to assign equal probabilities to each path but other ideas may be investigated. This would entail greater overheads if it were implemented on a router and therefore may be less desirable in practice. The authors intend to report this work in future papers.

5. REFERENCES


