Measurement of thickness of dielectric materials from microwave reflection data

Kailash Thakur and Wayne Holmes
Electromagnetic Sensing Competency
Imaging and Detecting Platform
Industrial Research Limited
24 Balfour Road, Parnell, Auckland
e-mail: k.thakur@irl.cri.nz

Subhas Chandra Mukhopadhyay
Institute of Information Sciences and Technology
Massey University (Turitea)
Palmerston North

Abstract

This paper presents a technique for the measurement of thickness of dielectric material using microwave. The measurements are carried out for the dielectric backed by metal reflector. The reflection coefficients at X-band frequencies are measured using a focussed microwave system so that a plane wave approximation can be applied. The results are presented for seven different samples of acrylic sheets backed by metal reflector. It is found that the microwave measurement done at a single frequency generates a number of solutions for the thickness. Hence measurements must be carried out at more than one frequency to get the correct answer for the thickness of the dielectric.

Keywords: plane dielectric, thickness, microwave, permittivity, plane wave, focusing antenna

1 Introduction

This paper presents a simple technique to estimate the thickness of dielectric materials using microwave. We have been involved in the development of thickness measurement system for dielectrics using microwave for past few years [1-3]. A mathematical solution for such measurements is usually an inverse technique. In the past, we have used nonlinear least squares regression [1-3], which generates an overall average value of thickness from a number of measurements.

This paper describes a technique which is useful for a quick estimation of the thickness for low loss or loss free dielectrics. It does not need a large number of data as required in the nonlinear least squares regression [1-3]. Moreover, this technique could be implemented in a low cost system. The system can be portable and hence quite useful for field measurements.

2 Measurement Principle

The dielectric sheet of thickness, *d* and permittivity, ε_r is placed on the metal surface as shown in Fig. 1. The plane microwave incident upon the sheet is reflected back and obeys the Fresnel's equations.

The Fresnel's equations for the reflection coefficient of a plane wave incident on the target medium backed by a highly reflective plane can be written as

$$\Gamma_\perp = \frac{j\mu_r \cos\theta_i \tan[k_d \cdot (\mu_r\varepsilon_r - \sin^2\theta_i)^{\frac{1}{2}}] - (\mu_r\varepsilon_r - \sin^2\theta_i)^{\frac{1}{2}}}{j\mu_r \cos\theta_i \tan[k_d \cdot (\mu_r\varepsilon_r - \sin^2\theta_i)^{\frac{1}{2}}] + (\mu_r\varepsilon_r - \sin^2\theta_i)^{\frac{1}{2}}} \tag{1}$$

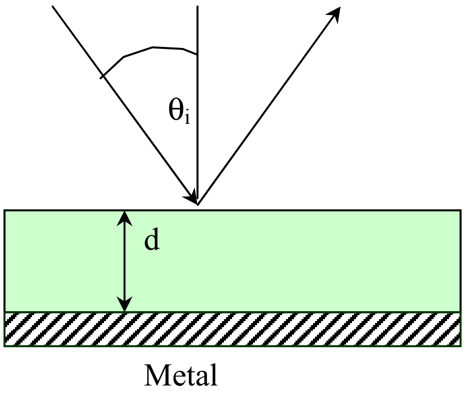

Figure 1: Experimental setup for the measurement of thickness

If the angle of incidence, θ_i is set to zero, we have the reflection coefficient for $\mu_r = 1$,

$$R = \frac{\frac{j}{\sqrt{\varepsilon_r}}\tan\left(kd\sqrt{\varepsilon_r}\right) - 1}{\frac{j}{\sqrt{\varepsilon_r}}\tan\left(kd\sqrt{\varepsilon_r}\right) + 1} \qquad (2)$$

Equation (2) can be solved to find the thickness of dielectric in terms of reflection coefficient, R, permittivity ε_r and wave vector in free space, k_o.

$$d = \frac{1}{k_0\sqrt{\varepsilon_r}}\left(\tan^{-1}\left[-j\sqrt{\varepsilon_r}\left(\frac{1+R}{1-R}\right)\right] + n\pi\right) \qquad (3)$$

where n is an integer and can provide a periodic solution for thickness, d. The problem with the solution of equation (3) is that the imaginary component of thickness, d is not always zero and hence does not provide a good solution. Another problem is that there are several solutions for thickness, d because of the periodic nature of the function. This problem can be resolved by using measurements at several frequencies using the more generalised and sophisticated algorithm [1].

However, for many commercial applications a quick and rough estimate might be needed using a low cost two or three frequency sensor. For this purpose, we have developed an algorithm, to solve for the thickness and permittivity of the dielectric backed by metal plate, using an iterative procedure. The strict requirement is that the sensor must be calibrated for the measurement of reflection coefficients on the plane of measurement.

3 Experiment and Results

In this measurement, we have used a focussing horn antenna connected to a six port detection system to measure the reflection coefficients. The calibration was carried out on the focal plane of the antenna. These measurements were carried out at 16 different frequencies ranging from 8.5 GHz to 10GHz.

The following algorithm is used for the computation of thickness, d from the measured reflection coefficients:

1. start with an initial value of permittivity, ε_r
2. calculate d using equation (3)
3. reduce the imaginary part of d by a small factor, 1.01 (say)
   imag (d) = imag (d)/1.01
4. calculate ε_r using

$$\sqrt{\varepsilon_r} = \frac{j\tan(k_0 d\sqrt{\varepsilon_r})}{\left(\frac{1+R}{1-R}\right)}$$

5. Go back to 2 until the difference between initial and final values of real component of ε_r is less than a predefined tolerance value

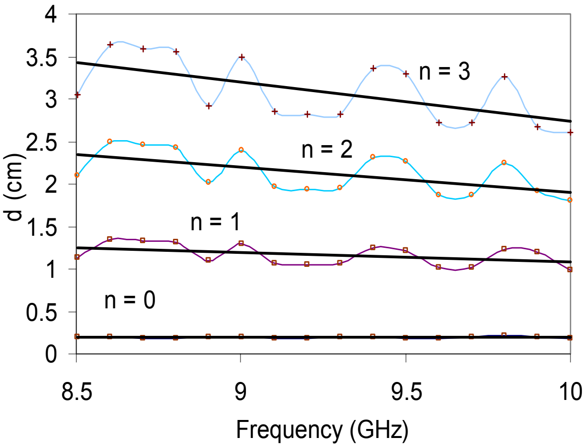

Figure 2: Plot of simulated values of thickness, d of a 0.2 cm thick acrylic sheet measured using equation (3) for different values of n. Only the results for $n = 0$ are acceptable in this case.

The results of the measurements for seven samples of different thicknesses of acrylic sheet are shown in Table 1.

The simulated results for thickness, d for four samples of acrylic sheet using the above algorithm are shown in Figures 2 to 5 for different values of n. The measured thickness of the sample varies from 0.2 cm to 2.01 cm and hence only one particular value of n is acceptable in each case for a correct result. It is interesting to note in Figure 2 that for $n = 0$, the value of thickness, d is almost constant around $d = 0.2$ cm. However, for $n > 0$, the deviation in the value of d increases and also the running average value of d decreases with increasing frequency. Similar results are seen in Figure 3. In Figure 4, the slope of the curve corresponding to $n = 1$ is the minimal whereas the slope is positive for $n = 0$ and negative for $n > 1$. In Figure 5, the slope of the curve corresponding to $n = 2$ is the minimal whereas the slope is positive for $n = 0$ and $n = 1$ and negative for $n = 3$.

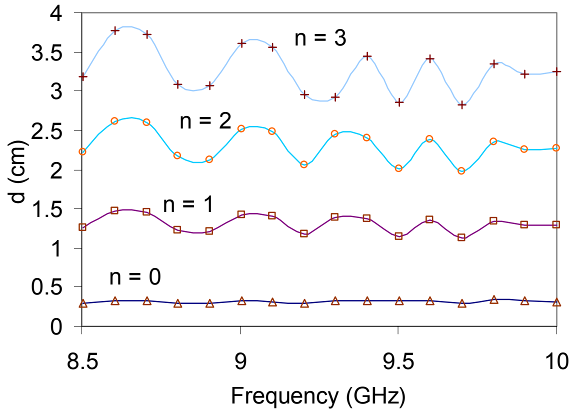

Figure 3: Plot of simulated values of thickness, d of a 0.32 cm thick acrylic sheet measured using equation (3) for different values of n. Only the results for $n = 0$ are acceptable in this case.

These observations form the basis for discrimination between the acceptable and not acceptable solutions of the periodic function (3), ie, the value of thickness, d measured at more than one frequency should be the same. Moreover, the value of dielectric constant generated by this algorithm is constant for the acceptable value of n and varies considerably for other values of n. This forms another possible way for the discrimination of acceptable and unacceptable solutions of equation (3).

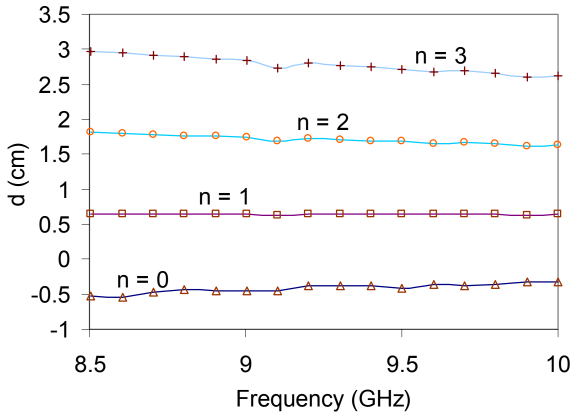

Figure 4: Plot of simulated values of thickness, d of a 0.6 cm thick acrylic sheet measured using equation (3) for different values of n. Only the results for n = 1 are acceptable in this case.

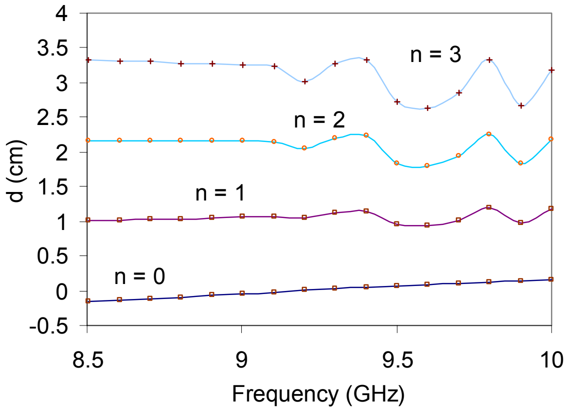

Figure 5: Plot of simulated values of thickness, d of a 2.01 cm thick acrylic sheet measured using equation (3) for different values of n. Only the results for n = 2 are acceptable in this case.

Table 1: Measured values of thickness and permeability of acrylic sheet in the X-band

n	***d*/cm (meas)**	***d*/cm eq. 3 real**	***d*/cm eq. 3 imaginary**	***ε'***	***ε''***
0	0.2	0.19	-0.003	2.182	0.000
0	0.2	0.19	0.003	2.645	-0.295
0	0.32	0.31	0.000	2.633	-0.122
1	0.62	0.64	0.000	2.317	-0.007
1	0.62	0.64	-0.006	2.339	-0.031
1	0.95	1.02	-0.004	2.328	-0.009
1	0.95	0.99	-0.005	2.459	-0.013
1	1.41	1.41	0.016	2.650	-0.117
1	1.41	1.42	-0.008	2.601	0.000
2	2.01	2.09	-0.011	2.480	-0.006
3	2.96	3.04	-0.015	2.518	-0.009
3	2.96	2.93	-0.016	2.776	-0.004

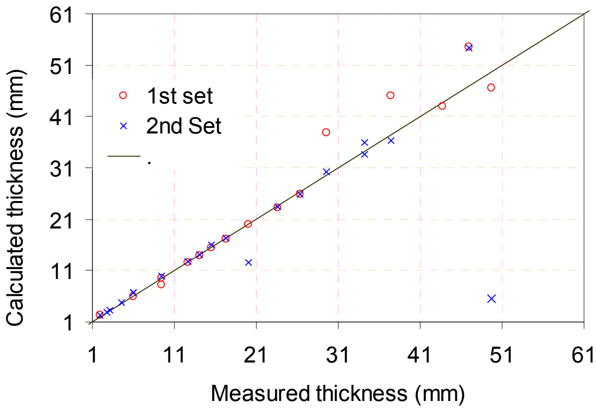

Figure 6: Measured and calculated thickness of various acrylic sheets.

We have also repeated two measurements for various acrylic samples. The results are presented in Figure 6 for the thickness of the acrylic samples. Only the acceptable solution for *n* has been chosen. The plot of the measured and estimated values of thickness in figure 6 shows the validity of the present technique. However, there are a few outliers also in the results which might be due to the wrong choice of *n*.

A wrong choice for *n* can be easily made in such studies, which is due the inherited nature of the inverse problem. We have found a simpler way to estimate the permittivity of the planar sample without taking the inverse. Consider two planar dielectrics of same material and of thicknesses, d_1 and d_2 so that the sum of the thickness is d. If both dielectrics are placed on the metal surface and the Reflection coefficient R is measured, we get

$$\Gamma = \frac{1+R}{1-R} = \frac{j.\tan(k.d.\sqrt{\varepsilon_r})}{\sqrt{\varepsilon_r}} \qquad (4)$$

The reflection coefficients, R_1 and R_2 are also measured for individual dielectrics of thickness, d_1 and d_2, ie

$$\Gamma_1 = \frac{1+R_1}{1-R_1} = \frac{j.\tan(k.d_1.\sqrt{\varepsilon_r})}{\sqrt{\varepsilon_r}} \qquad (5)$$

and

$$\Gamma_2 = \frac{1+R_2}{1-R_2} = \frac{j.\tan(k.d_2.\sqrt{\varepsilon_r})}{\sqrt{\varepsilon_r}} \qquad (6)$$

since

$$d = d_1 + d_2 \qquad (7)$$

From equations (4) and (7) we get

$$\Gamma = \frac{j.\tan\left(k.(d_1+d_2).\sqrt{\varepsilon_r}\right)}{\sqrt{\varepsilon_r}} \qquad (8)$$

or

$$\Gamma = \frac{j}{\sqrt{\varepsilon_r}} \frac{\left(\tan(k.d_1.\sqrt{\varepsilon_r}) + \tan(k.d_2.\sqrt{\varepsilon_r})\right)}{1 - \tan(k.d_1\sqrt{\varepsilon_r})\tan(k.d_2\sqrt{\varepsilon_r})} \qquad (9)$$

or

$$\Gamma = \frac{\Gamma_1+\Gamma_2}{1 - \frac{\sqrt{\varepsilon_r}\Gamma_1}{j}.\frac{\sqrt{\varepsilon_r}\Gamma_2}{j}} = \frac{\Gamma_1+\Gamma_2}{1+\varepsilon_r\Gamma_1\Gamma_2} \qquad (10)$$

From eq. (10) we get

$$\varepsilon_r = \frac{\Gamma_1+\Gamma_2-\Gamma}{\Gamma\Gamma_1\Gamma_2} \qquad (11)$$

We have carried out several measurements for several samples of acrylic of different thicknesses and their pairs. The results for dielectric constant and loss factor of acrylic sheet are shown in Figures 7 and 8. These results are close to the experimental values shown as a line. The discrepancy between the computed and experimental values may be attributed to the experimental error. Moreover, the samples taken from different supplier may not have the same dielectric constant.

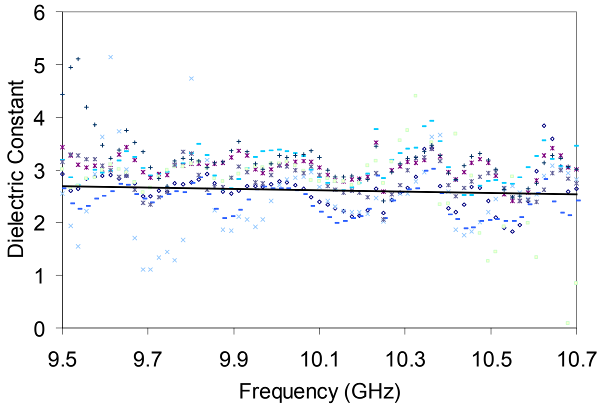

Figure 7: Computed values of dielectric constant from eq. (11) for samples for several pairs of acrylic sheets

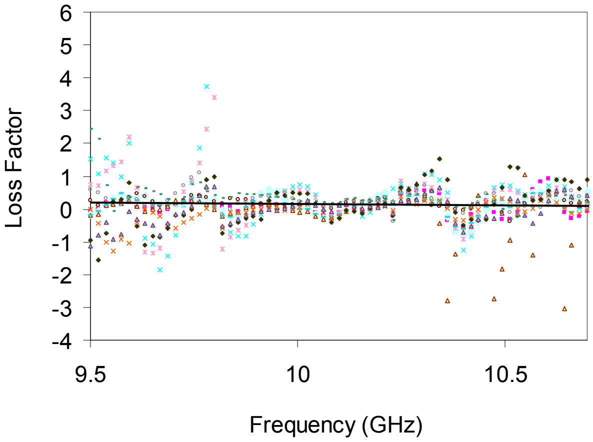

Figure 8: Computed values of loss factor from eq. (11) for samples for several pairs of acrylic sheets

4 Summary

A microwave measurement carried out at a single frequency will generate a number of solutions for the thickness. This makes it difficult to select the correct solution. Hence the measurement must be carried out at more than one frequency to get the correct answer for the thickness of the dielectric. If there is a strong dispersion in the dielectric, the technique will fail because there will be different answers for permittivity at different frequencies which may not be able to generate a single value for thickness, *d* at all frequencies. Measurement of thickness using microwaves in the presence of strong dispersion in the material can be carried out using the non-linear least squares regression technique described in [1]. The present algorithm tested here can be applied to the measurement of the thickness of dielectric layers backed by strong reflector eg, layer of ice on water, depth of metallic objects buried in soil. This paper presents results of our preliminary study and more investigation is needed to improve the calibration procedure of the focussing system to improve the technique.

5 Acknowledgements

The authors thank Industrial Research Limited for financial support through their Research and Development Funnel projects. Financial help from the Foundation for Research Science and Technology, New Zealand is also acknowledged.

6 References

[1] Thakur K. P., Holmes, W. S. "Non-contact measurement of moisture in layered dielectrics from microwave reflection spectroscopy using an inverse technique", *IEEE Transactions on Microwave Theory and Techniques, 52(1), pp. 76-82, January* (2004).

[2] Thakur K. P. "Moisture measurement in Multi-Layered system", Book Chapter in "*Electromagnetic Aquametry*", Ed. K. Kupfer, published by Springer-Verlag, Germany, pp. 113-133. (2005).

[3] Thakur K., Chan K L, Holmes W. S., "Thickness and permittivity of inhomogeneous dielectrics from microwave reflection spectroscopy using an inverse technique", *Fifth International Conference on Electromagnetic Wave Interaction with water and Moist Substances, International Society for Electromagnetic Aquametry (ISEMA),* Rotorua, New Zealand, pp. 118-128, March (2003).