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Investigation of classical and non-classical sensor calibration approaches

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Abstract

Self-validation of sensors is often based on model parameters estimation by calibration. In order to "repair" the sensor's behavior by correction of the model parameters, it is important to identify which properties are the reasons for losses of accuracies failures. In this contribution a general method for the investigation of classical and so-called non-classical calibration schemes (with a number of reference points of the measuring value being fewer than parameters to estimate) is presented. Based on evaluation of the functional determinant, i.e. the determinant of the Jacobi matrix, it is examined whether variations of working points at reference conditions instead of reference points of the measurement value could be used for the parameter estimation. In principle, with the presented method all kinds of automatic self-calibration of sensors can be analyzed. As examples, the calibration of two different distance sensors as well as the zero-point calibration of semiconductor gas sensors and the self-calibration of an air-flow sensor are discussed.

Keywords: sensor self-calibration, SEVA sensors, Jacobi matrix, uncertainty, GUM

1 Introduction

For most practical applications, simplicity (costs) and lifetime are generally higher valued than absolute accuracy. It is desired, that during their lifetime the devices work properly without maintenance. In order to avoid system failures, the validity of sensor output data in accordance to the specification should be monitored with built-in "smart" monitoring features.

A sensor will be regarded faulty, if its measurement deviation exceeds certain error boundaries, which are given by the uncertainty of the sensor. Numerous national and international efforts exist to evaluate the uncertainty of measurements (e.g. [1-4]). According to the "Guide to the Expression of Uncertainty in Measurement" (GUM) [5], the uncertainty of measurements is described by the sensor model $y = f(x_i)$, where y is the measurement value and x_i are the expectations of all input values and output signals (like signal display), influencing variables and parameters. The measurement results are given by $Y = y \pm U$, with expanded uncertainty U

$$U = u_y \cdot k_p, \quad (k_p \text{ - coverage factor}) \qquad (1a)$$

$$u_y = \left\{ \sum_i \left(\frac{\partial f}{\partial x_i}\right)^2 u_{x_i}{}^2 + 2 \cdot \sum_i \sum_j \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u_{x_i x_j} \right\}^{1/2} \quad (i \neq j) \qquad (1b)$$

The uncertainty values u_{x_i} and $u_{x_i x_j}$ are measured or estimated from the probability density function (PDF). The uncertainty of measurements is therefore significantly dependent on changes of expectations and uncertainties of the model parameters. This is a direct consequence from the GUM guidelines and has substantial effect on the evaluation of measurement results, as will be shown in the sequel.

As a simple example may serve a (almost linear) temperature sensor with the model function $R = R_0(1 + \alpha \cdot \vartheta)$, resp. $\vartheta = \frac{R - R_0}{\alpha \cdot R_0}$, like suggested by GUM. Both the expectation and the uncertainty of the measured ϑ calculated from the measured resistance R depend on the parameter values R_0 and α. Assuming independent parameters, with

$$\frac{\partial \vartheta}{\partial R} = \frac{1}{\alpha \cdot R_0} \; ; \; \frac{\partial \vartheta}{\partial R_0} = -\frac{R}{\alpha \cdot R_0{}^2} \; ; \; \frac{\partial \vartheta}{\partial \alpha} = \frac{1}{\alpha^2}\left(1 - \frac{R}{R_0}\right) \qquad (2)$$

the uncertainty u_ϑ of ϑ is expressed as:

$$u_\vartheta = \left\{ \left(\frac{1}{\alpha \cdot R_0} u_R\right)^2 + \left(\frac{R}{\alpha \cdot R_0{}^2} u_{R_0}\right)^2 + \left(\frac{1}{\alpha^2}\left(1 - \frac{R}{R_0}\right) u_\alpha\right)^2 \right\}^{1/2} \qquad (3)$$

If the uncertainty u_R of the measurement of resistance R is small enough, then u_ϑ is determined almost only by the expectations and, first of all, by the parameter uncertainties u_{R_0} and u_α. This general statement is very important for the description of the behavior of a sensor, especially during operation. It says that a change of the parameter values, e.g. due to dust or ageing, changes both the expectation and the uncertainty of measurement.

The problem is how to estimate changes of parameter values during operation. Many approaches have been proposed using redundancy, changes of input values, or superposition with a reference, see e.g. [1-4, 6-9]. Other approaches use reference points given by the system or implement new reference points of the measurement value (e.g. [10,11]), or use multiple models with additional knowledge of specific parameters [12] or known dependencies of the model's working point [13-16].

With respect to the various approaches for self-calibration, a general method would be desired to give an indication which approach offers the best cost-benefit relation for the specific problem.

2 Sensor self-calibration

2.1 Evaluation of the functional determinant

Self-calibration of sensors often deals with the observation of parameter changes in the model function of the sensor behavior. The common procedure is to calibrate the sensor with at least as much reference points of the measuring value as there are parameters to observe. In order to "repair" the sensor's behavior by correction, foremost the reasons for failures are to be identified. For this purpose, the model parameters are classified in type-specific and exemplary-specific and whether they are stable in time or not [11]. During operation, only the exemplary-specific parameters unstable in time are to be estimated.

A considerable part of costs for calibration built by making of reference points of the measurement value. Naturally, the re-calibration effort decreases with the number of parameters. But, the costs would further decrease significantly, if calibration would be possible with a lower number of references of the measurement value.

In this contribution, an universal method to analyze approaches for sensor calibration is presented. The basic idea is to consider the functional determinant (FD), i.e. the determinant of the Jacobi matrix as a measure for the resolvability of the parameter values from the set of equations describing the calibration procedure [1]. However, using this proposed method, the possibilities discussed in [1] are principally expanded. Different approaches for the calibration process can be investigated, using e.g. reference points of the measurement value (as will be shown on behalf of the introductory example of a linear temperature sensor as well as in [2,7,8,10]), implementing additional reference values like [1,2] shown on examples of a distance sensor or introducing and increasing the number of reference working points used (see example in Section 3 and [11,13,14,15]).

Generally, for a solution, the functional determinant must be unequal to zero:

$$FD(x_i; x_1, \ldots, x_n) = \begin{vmatrix} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial g_n}{\partial x_1} & \cdots & \frac{\partial g_n}{\partial x_n} \end{vmatrix} \neq 0 \tag{4}$$

with the general sensor function $g_j = f(x_i; x_j) - y_j$ at measurement points $(x_j; y_j)$ with parameters x_i. Hence, the parameters estimation must be conducted such that this condition will be satisfied.

To introduce in the matter, the simple temperature sensor from above will be considered: At reference points $(R_j; \vartheta_j)$ the general sensor function is

$$g_{1,2} = \frac{R_{1,2} - R_0}{\alpha \cdot R_0} - \vartheta_{1,2} = 0 \tag{5}$$

This system of equations is solvable for the parameters α and R_0 if (see Eqs. (2), (4))

$$\frac{D(g_1; g_2)}{D(\alpha, R_0)} = \begin{vmatrix} \frac{\partial g_1}{\partial \alpha} & \frac{\partial g_1}{\partial R_0} \\ \frac{\partial g_2}{\partial \alpha} & \frac{\partial g_2}{\partial R_0} \end{vmatrix} = \frac{1}{\alpha^3 R_0^2}(R_1 - R_2) \neq 0 \tag{6}$$

This is satisfied for $R_1 \neq R_2$.

The inverse general sensor function (which often is easier to use) is

$$g_{1,2}^{inv} = R_0 + \alpha \cdot \vartheta_{1,2} \cdot R_0 - R_{1,2} = 0 \tag{7}$$

with

$$\frac{D\left(g_1^{inv}, g_2^{inv}\right)}{D\left(\alpha, R_0\right)} = R_0(\vartheta_1 - \vartheta_2) \neq 0 \tag{8}$$

for $\vartheta_1 \neq \vartheta_2$ and hence $R_1 \neq R_2$. It would also be possible to use several reference points and to apply statistical methods to compensate for stochastical deviances, but this is not considered here.

From this simple example it is obvious that for the estimation of every parameter, one reference point of the measuring value is needed. Here, this is called 'classical' calibration.

2.2 Non-classical self-calibration

Generally, it is costly to implement and evaluate additional reference points. Moreover, in many real-world applications, for a proper re-calibration of the sensor, the measurement value has to be somehow "disconnected" from the sensor input and the reference value has to be connected in place.

If the reference can be "switched on" as a superposition, the solution becomes easier: e.g. an additional mass is used for scales, or a additional displacement for a distance sensor [2]. In these cases, the measurement value have not to be "disconnected" from the input during calibration, saving considerable costs. For investigation of the question, which parameters can be observed this way, the functional determinant of the sensor model will be again used.

As a first example, the calibration of a linear distance sensor with the model $y = k \cdot x + n$ is described. Measuring the unknown distance *x* and an additional reference Δx_1, the following equations for 2 output values y_0 and y_1:

$$\begin{aligned} g_0 &= k \cdot x + n - y_0 = 0 \\ g_1 &= k \cdot (x + \Delta x_1) + n - y_1 = 0 \end{aligned} \tag{9}$$

deliver

$$\frac{D(g_0, g_1)}{D(k, x)} = \begin{vmatrix} x & k \\ x + \Delta x_1 & k \end{vmatrix} = -k \cdot \Delta x_1 \neq 0 \tag{10}$$

and

$$\frac{D(g_0, g_1)}{D(n, x)} = \begin{vmatrix} 1 & k \\ 1 & k \end{vmatrix} = 0 \tag{11}$$

From (10) and (11) it becomes clear, that with this approach *x* and the parameter *k* can be estimated, but not *n*, since the matrix (11) is singular. Also the attempt to use a second superposed reference Δx_2 will not solve the problem (estimation of *x*, *k* and *n*), since from $g_2 = k \cdot (x + \Delta x_2) + n - y_2 = 0$ results:

$$\frac{D(g_0, g_1, g_2)}{D(k, n, x)} = 0 \tag{12}$$

However, introducing an additional reference point at $x = x_3$ yields $g_3 = k \cdot x_3 + n - y_3 = 0$ and

$$\frac{D(g_0, g_1, g_3)}{D(k, n, x)} = k \cdot \Delta x_1 \tag{13}$$

and hence $\{x,k,n\}$ can be estimated.

Figure 2 shows a sketch of such a distance sensor with full self-calibration capability: the cone end CE which registers the thickness *x* of a workpiece is pressed on by a spring S1. If the electromagnet EM1 is switched on, the sensor element SE "feels" the reference value x_3, and if the electromagnet EM2 is switch on, the measured value *x* is displaced by the additional distance Δx_1.

In a second example to verify the applicability of the described approach for a simple evaluation of calibration strategies, the model of the distance sensor is assumed to be nonlinear with $y = \frac{a}{1 + b \cdot x}$. Using an additional displacement Δx_1 as a reference, from

$$\begin{aligned} g_0 &= \frac{a}{1 + b \cdot x} - y_0 = 0 \\ g_1 &= \frac{a}{1 + b \cdot (x + \Delta x_1)} - y_1 = 0 \end{aligned} \tag{14}$$

yields

$$\frac{D(g_0, g_1)}{D(b_1, x)} = \begin{vmatrix} \frac{-ax}{(1+bx)^2} & \frac{-ab}{(1+bx)^2} \\ \frac{-a(x+\Delta x_1)}{(1+b(x+\Delta x_1))^2} & \frac{-ab}{(1+b(x+\Delta x_1))^2} \end{vmatrix} = \frac{-ab\Delta x_1}{(1+bx)^2(1+b(x+\Delta x_1))^2} \neq 0 \tag{15}$$

Hence, the values *x* and *b* can be estimated.

With a reference point at $x = x_3$ the sensor equation becomes $g_3 = \frac{a}{1 + b \cdot x_3} - y_3 = 0$ and the determinant will be

$$\frac{D(g_0, g_1, g_3)}{D(a, b, x)} = \frac{a^2 b\, \Delta x_1 \left[\left\{ b^2 (x - x_3) - b \right\} x_3 - 1 \right]}{(1+bx)^2 (1+bx_3)^2 (1 + b(x + \Delta x_1))^2} \neq 0 \tag{16}$$

what means, that now the unknown measurement value *x* and the model parameters *a* and *b* can be estimated.

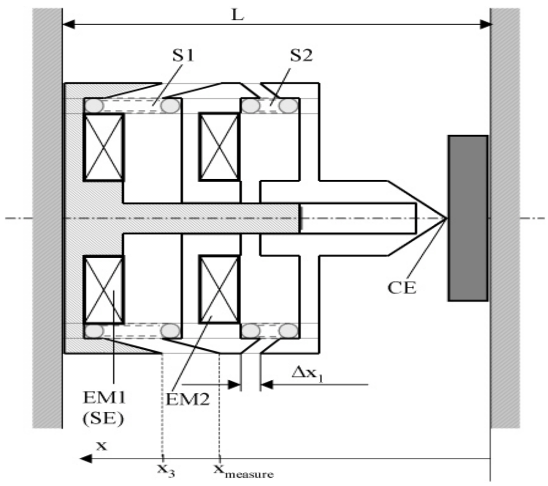

Fig.1: *Sketch of the proposed distance sensor with self-calibration capability*

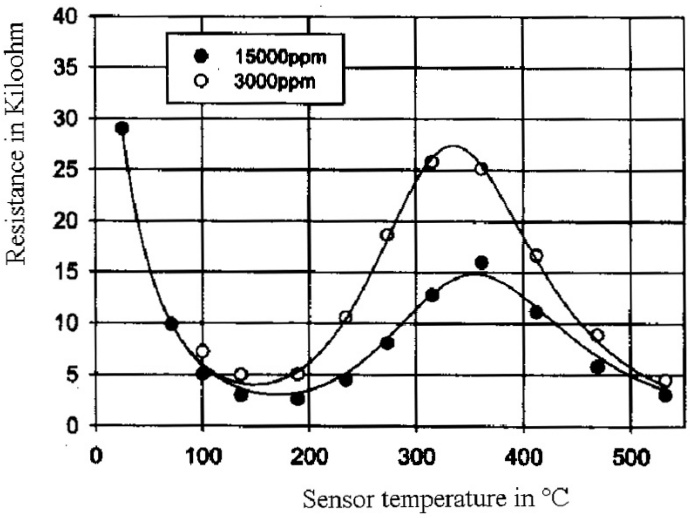

Fig. 2: *Typical resistance of a semiconductor gas sensor as a function of temperature with humidity as the parameter: Measured points and modeled function of of the sensor resistance in dependence of the temperature and humidity for the gas concentration x = 0 ppm [15]*

3 Advanced examples

Up to this point, a kind of composition of the 'classical' calibration using reference points of the measurement values (x_3 in the previous example) and a 'non-classical' approach using a superposition with additional values (Δx_1) has been described.

In the following examples, instead of reference points of the measuring value, working points at reference conditions (so called 'reference working points') are used. This avoids any 'classical' approach being the state of the art and hence can be called the 'pure non-classical' sensor calibration. The applicability of this approach will be verified using the same method as described in the previous section.

3.1 Zero-point recalibration of semiconductor gas sensors

As a first example, the calibration of the zero-point of semiconductor gas sensors is examined. In a step-by-step procedure, three degrees of the new method are developed, reducing the calibration effort in every step.

If the gas concentrations are negligible, the humidity dependent zero-point of a semiconductor gas sensor has a characteristic as shown in Fig. 2. It can be described by [15,16]

$$G(x_W, T) = G_{0S} \cdot a(T) + G_{0W} \cdot b(T) \frac{x_W{}^q \cdot k_W(T)}{1 + x_W{}^q \cdot k_W(T)} \tag{17}$$

where x_w is the water concentration. $a(T)$, $b(T)$ and $k_w(T)$ are physically determined type-specific functions, which are positive and strongly monotone with absolutely temperature T. The area-dependent and hence exemplary-specific conductance's G_{os} and G_{ow} are generally not exactly determined in the manufacturing process and unstable in time and must therefore be monitored.

The simplest and classical method to estimate G_{os} and G_{ow} is to calibrate them at constant temperature and two water concentrations $x_{w,1}$, $x_{w,2}$ (i.e. at two reference points of the measurement value x_w), see Eq. (17). The calculation of FD is similar to Eq. (6).

A second approach is to consider only a single constant and known concentration x_w at two temperatures T_1 and T_2. Hence, only one reference point of the measurement value at two working points is used to estimate two parameters. The calibration procedure would be as follows:

$$g_{1,2} = G_{0S} \cdot a(T_{1,2}) + G_{0W} \cdot b(T_{1,2}) \frac{x_W{}^q \cdot k_W(T_{1,2})}{1 + x_W{}^q \cdot k_W(T_{1,2})} - G_{1,2} = 0 \tag{18}$$

$$\frac{\partial g_{1,2}}{\partial G_{0S}} = a(T_{1,2}) = a_{1,2}\,,$$

$$\frac{\partial g_{1,2}}{\partial G_{0W}} = b(T_{1,2})\frac{x_W{}^q \cdot k_W(T_{1,2})}{1 + x_W{}^q \cdot k_W(T_{1,2})} = b_{1,2} \cdot X_{W1,2} \qquad (19)$$

$$\begin{vmatrix} \frac{\partial g_1}{\partial G_{0S}} & \frac{\partial g_1}{\partial G_{0W}} \\ \frac{\partial g_2}{\partial G_{0S}} & \frac{\partial g_2}{\partial G_{0W}} \end{vmatrix} = \begin{vmatrix} a_1 & b_1 \cdot X_{W1} \\ a_2 & b_2 \cdot X_{W2} \end{vmatrix} \qquad (20)$$

$$= a_1 \cdot b_2 \cdot X_{W2} - a_2 \cdot b_1 \cdot X_{W1} \neq 0$$

The effort of calibration is obviously much lower compared to the classical method.

A third method is proposed, using three temperatures (i.e. three working points) T_1, T_2, T_3. In this case, even an unknown water concentration x_w can be estimated from the sensor model, provided it is sufficiently constant during the time of calibration. Additionally to Eqs. (18,19) also the derivation of the general sensor function to x_w is to consider:

$$\frac{\partial g_j}{\partial x_W} =$$

$$G_{0W} \cdot b(T_j)\frac{x_W{}^q \cdot k_W(T_j)}{1 + x_W{}^q \cdot k_W(T_j)} \cdot \left[1 - \frac{x_W{}^q \cdot k_W(T_j)}{1 + x_W{}^q \cdot k_W(T_j)}\right] =$$

$$G_{0W} \cdot b_j \cdot Y_{Wj} \qquad , \quad j = 1,2,3 \qquad (21)$$

In this case, a solution of the functional determinant (Eq. (14)) is provided if the temperatures are unequal:

$$\begin{vmatrix} \frac{\partial g_1}{\partial G_{0S}} & \frac{\partial g_1}{\partial G_{0W}} & \frac{\partial g_1}{\partial x_W} \\ \frac{\partial g_2}{\partial G_{0S}} & \frac{\partial g_2}{\partial G_{0W}} & \frac{\partial g_2}{\partial x_W} \\ \frac{\partial g_3}{\partial G_{0S}} & \frac{\partial g_3}{\partial G_{0W}} & \frac{\partial g_3}{\partial x_W} \end{vmatrix} = \begin{vmatrix} a_1 & b_1 \cdot X_{W1} & G_{0W} \cdot b_1 \cdot Y_{W1} \\ a_2 & b_2 \cdot X_{W2} & G_{0W} \cdot b_2 \cdot Y_{W2} \\ a_3 & b_3 \cdot X_{W3} & G_{0W} \cdot b_3 \cdot Y_{W3} \end{vmatrix} =$$

$$G_{0W} \cdot \begin{bmatrix} X_{W1}(a_3 b_1 b_2 Y_{W2} - a_2 b_1 b_3 Y_{W3}) \\ + X_{W2}(a_1 b_2 b_3 Y_{W3} - a_3 b_1 b_2 Y_{W1}) \\ + X_{W3}(a_1 b_2 b_3 Y_{W2} - a_2 b_1 b_3 Y_{W1}) \end{bmatrix} \neq 0 \qquad (22)$$

In Fig.3, the results of this calibration method are shown for three different gas sensors verifying the applicability of the new approach [16]. The result is especially interesting for long-time applications.

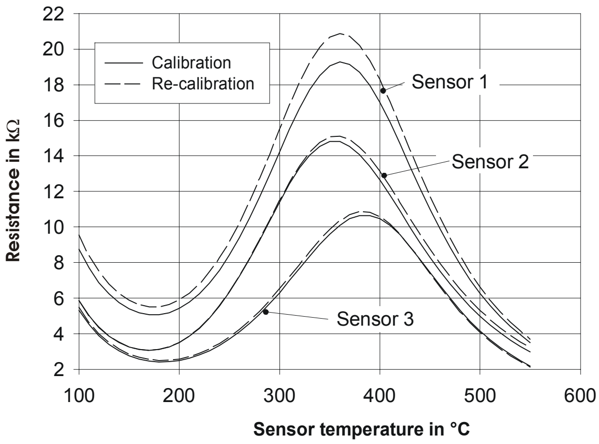

Fig. 3. *Results from the re-calibration of the zero-point of semiconductor gas sensors at humidity level assumed as unknown (15000 ppm in this case).*

3.2 Self-calibrated air flow sensor

The second example deals with the self-calibration of an air flow sensor with a self heated PTC as active device. The temperature-dependent electrical resistance *R(T)* is given by

$$R(T) = R_{T0} \cdot e^{B\frac{T - T_C}{T}} + R_0 \qquad (23)$$

In the steady state between electrical and thermal power P with the thermal resistance R_{th}, $R(P)$ will be

$$R(P) = R_{T0} \cdot e^{B\frac{P \cdot R_{th} + T_E - T_C}{P \cdot R_{th} + T_E}} + R_0 \qquad (24)$$

R_{th} in dependence of the air velocity v can be described as [11]

$$R_{th} = R_{th,S} + \\ + \frac{1}{2\pi L\lambda}\ln\frac{D}{d} + \frac{1}{a_0\left(1 + a_1\left(\frac{D}{d}\right)^{0.5}\right)^2 + b_0\sqrt{\frac{D}{d}v + b_1\left(\frac{D}{d}\right)^{1.6} v^{1.6}}} \qquad (25)$$

Three parameters changing over time have to be monitored: the value of the diameter D of the sensor (by soiling or contamination of its surface) and R_{T0} and R_0 (by aging). Under the conditions that $v=0$ and the temperature T_E of the environment is known, for different values P_i of the thermal power, the general sensor function can be written as

$$g_i = R_{T0} \cdot e^{B\frac{P_i \cdot R_{th} + T_E - T_C}{P_i \cdot R_{th} + T_E}} + R_0 - R(P_i) = 0 \qquad (26)$$

and hence

$$\frac{D(g_1, g_2, g_3)}{D(R_{T0}, R_0, D)} \neq 0 \qquad (27)$$

Fig.4 shows the measured air velocity as a function of the air velocity for a new sensor and a contaminated sensor with and without the described model-based correction, showing that the contamination is detected and the errors are compensated for almost perfectly.

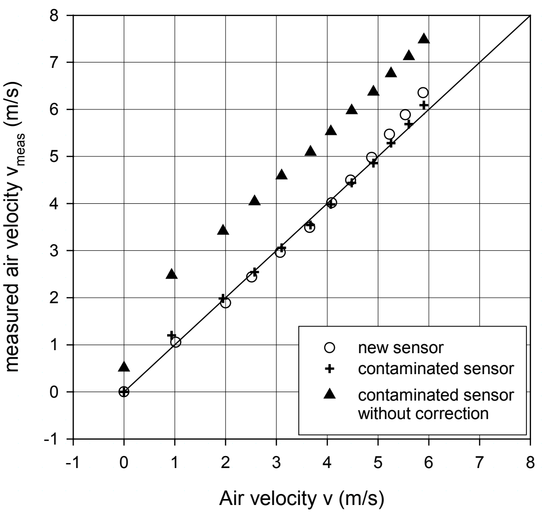

Fig. 4. *Measured air velocity against real air velocity with and without the described model-based correction*

4 Conclusion

According to the "Guide to the Expression of Uncertainty in Measurement" (GUM) changes of parameters and their uncertainties directly determine the measurement uncertainty. Therefore, in order to evaluate the uncertainty of measurements parameter expectations and uncertainties must be considered.

In this paper, a general methodology is developed to analyze approaches for a 'classical' and so called 'non-classical' sensor calibration. It is based on the examination of the functional determinant of the sensor model and uses working points at reference conditions (reference working points) instead of reference points of the measurement value. Hence, a significant reduction of the calibration effort is achieved.

The applicability of the methods is verified at 5 practical examples: a simple linear temperature sensor, a linear and nonlinear distance sensor and the advanced problems of zero-point re-calibration of semiconductor gas sensors and a self-calibrated air-flow sensor. Similarly, several other calibration methods can be addressed using the presented methodology.

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