1st International Conference on Sensing Technology
November 21-23, 2005 Palmerston North, New Zealand

Parameter Estimation of One-Axis Magnetically Suspended System with a Digital PID Controller

Lim Tau Meng, Cheng Shanbao, Chua Leok Poh
School of Mechanical and Aerospace Engineering
Nanyang Technological University, Singapore 639798
mtmlim@ntu.edu.sg, chen0181@ntu.edu.sg, mlpchua@ntu.edu.sg

Abstract

An electromagnetically suspended system can be regarded as a typical second order mass-spring-damping system. The system dynamic response and stability are somehow related to its PID (proportional, integral and derivative) controller feedback parameters. Although the integral feedback is largely used to eliminate its offset errors at lower frequency, the actual stiffness and damping coefficients related to PD control are vital for the accurate prediction of the suspended system responses during design stage. Numerous studies have attempted to estimate these coefficients directly or indirectly without much success. The problem is due to the frequency-dependent nature of the active magnetic bearing (AMB) system i.e. time lags in sensors, digital signal processing, amplifiers, filters, eddy current and hysteresis losses in the electromagnetic coils. This paper seeks to address this problem, by proposing a one-axis electromagnetic suspension system to simplify the measurement requirements and eliminate the possibility of control force cross-coupling capabilities. This would lead to a better understanding and a design platform for optimal vibration control strategy of suspended system. This is achieved by injecting Schroeder Phased Harmonic Sequences (SPHS), a multi-frequency test signal, to persistently excite all possible suspended system modes. By treating the system as a black box, the parameter estimation of the "actual" stiffness and damping coefficients in the frequency domain are realised experimentally. The digitally implemented PID controller also facilitated changes on the feedback gains, and this allowed numerous system response measurements with their corresponding estimated stiffness and damping coefficients.

Keywords: magnetic bearing, electromagnetic actuator, parameter estimation, PID controller, SPHS

1. Introduction

Active magnetic bearings have been receiving widespread applications in industry [1-6]. This is because they can support the rotor without contact, thus no friction and heat are generated and the need for lubrication is also eliminated. Another distinct advantage of AMB, is that its dynamic characteristics can be easily modified by changing the gains of its controller. This is because the stiffness and damping properties of magnetic bearings are closely related to the feedback controller parameters. R.D. Williams, *et al*. [1] studied on the effective characteristics of AMB with various feedbacks controller parameters. The feedback parameters were digitally implemented on an experimental rotor-bearing-system. However, the estimated equivalent stiffness and damping coefficients obtained are based on a theoretically derived frequency-dependent feedback controller transfer function. This method for estimating both the stiffness and damping coefficients raises a fundamental question on possible time lags in the digital signal processing, amplifiers, feedback sensors, cross-coupling capability and electromagnet's eddy current and hysteresis losses etc. in the system [7]. This problem is magnified by the cross-coupling capability of the AMB system in the orthogonal directions; which requires substantial displacement measurements in both the linear and angular directions at various locations of the plant [8]. In a practical plant, some measurement locations may not be accessible. Xie Z.Y. *et al*. [2] and Humphris, R.R. *et al*. [3] reported experimental results on influences of feedback control parameters on the dynamic responses of AMB system. However, only limited experimental results on rotor displacements with various feedback control parameters were reported. The stiffness and damping coefficients of magnetic bearings, which are important to predict rotor-bearing system response at the design stage, are not obtained [8][9].

In this paper, a digital PID electromagnetic suspension system is realized with one degree of freedom (DOF) motion. With different parameters (*Kp*, *Ki*, and *Kd*) of PID controller, actual stiffness and damping coefficients of the electromagnetic actuator are estimated in the frequency domain. It uses multi-frequency SPHS test signal [4] to persistently excite the mode of the suspended platform. The system excitation force/displacement transfer function is obtained experimentally. It is in this manner, the actual stiffness and damping coefficients of the electromagnetic actuator are obtained. Comprehensive relationships between digital PID controller gains versus stiffness and damping coefficients and responses are studied.

2. Principles

2.1. *Effective* Stiffness and Damping Coefficients of Electromagnetic Suspension System

The actuation force for a single DOF active magnetic suspension system [1] can be expressed by its mechanical restoring stiffness (K_{yy}) and current related stiffness (K_{iy}) respectively. In the control loop for the uncoupled one DOF motion, the simplified block diagram of the suspension system is shown in Figure 1

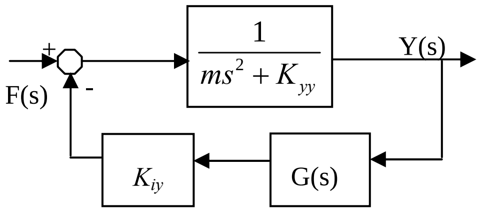

Figure 1: Block diagram of AMB system in one axis

The transfer function of the close-loop system can be expressed as:

$$\frac{Y(s)}{F(s)}=\frac{1}{ms^2+K_{yy}+K_{iy}G(s)} \quad \textbf{(1)}$$

The transfer function of equation (1) in the frequency domain can be expressed as:

$$\frac{Y(j\omega)}{F(j\omega)}=\frac{1}{-m\omega^2+K_{yy}+K_{iy}G(j\omega)} \quad \textbf{(2)}$$

where *Y(jω)* is the Fourier transform of the displacement *Y(s)*, and *F(jω)* is the Fourier transform of excitation force *F(s)*. It is well known that, magnetic bearing system can be modelled as a typical second order mass-spring-damping system. Therefore, the system equation of motion can be written as:

$$m\ddot{y}+ky+c\dot{y}=f_y, \qquad (k>0, c>0) \quad \textbf{(3)}$$

where *m, k* and *c* represent the rotor mass, stiffness and damping coefficients respectively. Equation (3) can be written in the frequency domain,

$$\frac{Y(j\omega)}{F(j\omega)}=\frac{1}{-m\omega^2+k+cj\omega} \quad \textbf{(4)}$$

Combining equation (2) and (4), we can obtain the effective stiffness and damping coefficients of the magnetically suspended system i.e. by separating the real and imaginary parts of equation (5).

$$k=K_{yy}+K_{iy}\,\mathrm{Re}(G(j\omega)) \qquad c=K_{iy}\frac{\mathrm{Im}(G(j\omega))}{\omega} \quad \textbf{(5)}$$

They are called "effective" or "equivalent" coefficients because they are not the actual coefficients associated with equation (3). And they are obtained using the theoretical frequency dependent PID controller's transfer function *G(jω)*. This *G(jω)* may exclude system lags due to digital signal processing, amplifiers, feedback sensor, cross-coupling capability and actuator losses etc.

2.2. *Actual* Stiffness and Damping Coefficients of Electromagnetic Suspension System

In the frequency domain, equation (3) can be expressed as:

$$F(j\omega)=-m\omega^2Y(j\omega)+kY(j\omega)+cj\omega Y(j\omega) \quad \textbf{(6)}$$

Equation (6) can be expressed as the transfer function of force/displacement:

$$\frac{F(j\omega)}{Y(j\omega)}=-m\omega^2+k+cj\omega \quad \textbf{(7)}$$

The real and imaginary parts of the Fast Fourier Transform of equation (7) are expressed as:

$$\left.\begin{aligned}\mathrm{Re}\left(\frac{F(j\omega)}{Y(j\omega)}\right)&=-m\omega^2+k\\ \mathrm{Im}\left(\frac{F(j\omega)}{Y(j\omega)}\right)&=\omega c\end{aligned}\right\} \quad \textbf{(8)}$$

The actual stiffness coefficient *k* and damping coefficient *c* of the electromagnetic actuator can be rewritten as:

$$\left.\begin{aligned}k&=m\omega^2+\mathrm{Re}\left(\frac{F(j\omega)}{Y(j\omega)}\right)\\ c&=\frac{\mathrm{Im}\left(\frac{F(j\omega)}{Y(j\omega)}\right)}{\omega}\end{aligned}\right\} \quad \textbf{(9)}$$

These coefficients in equation (9) are called "actual" coefficients of the single-axis suspension system of equation (3). This because they take into account of the known system mass *m* and are calculated directly from the measured force/displacement transfer function in the frequency domain. The measurements include all possible system time lags and losses, and therefore they truly represent the "actual" stiffness and damping coefficients of the magnetic bearings.

3. Experimental Result

The experimental setup for the frequency domain parameter estimation of the stiffness and damping coefficients of the electromagnetic suspension system is as shown in the schematic of Figure 2. The H-shaped rigid platform is suspended by the magnetic forces generated by a pair of E-frame electromagnetic actuators. Its coil currents are actively controlled by a digital PID controller via a PC using the dSPACE DS1103 controller board. A broadband SPHS [4] multi-frequency test signal with bandwidth of 1 to 200Hz and fundamental frequency of 1 Hz, is used to persistently excite all modes in the suspended system. This digital signal is calculated by Matlab software and fed to a shaker via the DS1103 DAC. An eddy current probe and force transducer are used to measure the displacements of the platform and its excitation forces respectively. The displacement and force signals are connected to the PC via the DS1103 ADC; where they are transformed into frequency-domain signals by a FFT software. The photo for the experimental setup in Figure 3 shows the 160mmx160mmx12mm and 0.92kg H-shaped platform being constrained to move only in the vertical direction by means of 4 sliding bearing with negligible friction. The air gap between the electromagnets and platform is set at 1mm. The DC bias premagnetization current for the electromagnets is 0.65A. Therefore, the mechanical stiffness of electromagnetic actuator, K_{yy}=-53.3kN/m, and the current related stiffness, K_{iy}=47.19N/A. The sampling rate of our digital PID close loop control system is set at 4 kHz to avoid signal aliasing and provide adequate control bandwidth.

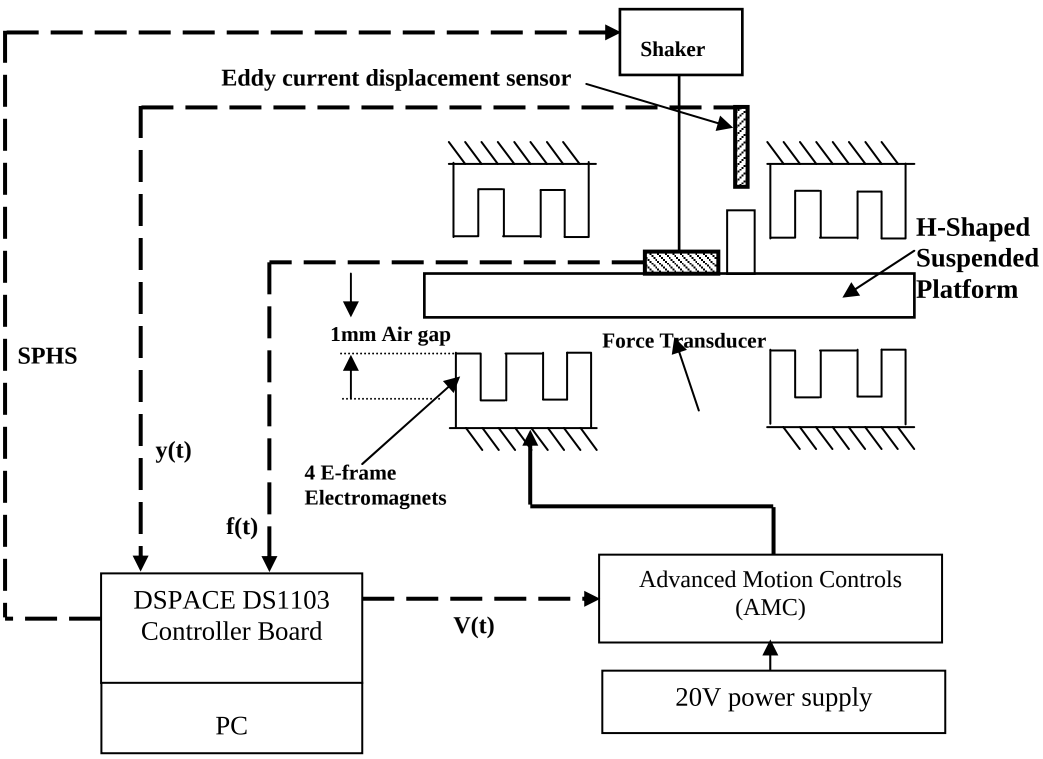

Figure 2: Instrumentation arrangement of experiment

3.1. Estimation of Effective and Actual Stiffness and Damping coefficients

Using equation (5), the effective stiffness and damping coefficients of the electromagnetic actuator in this experiment are as shown in Figure 4. It can be seen from Figure 4 that the effective stiffness and damping coefficients of the electromagnetic actuator are frequency dependent and smooth - due to the theoretically derived control $G(j\omega)$. This graph is also consistent with the findings of R.D. Williams, *et al*.[1]. However, their damping coefficients obtained decrease in the high frequency range. This may be due to the double derivative feedback (PIDD) control employed in their AMB system; while we are using a PID controller in our experiment. The actual stiffness and damping coefficients in Figure 5 are obtained at discrete excitation frequency. Signal averaging of 10 periods and median filtering are used to smooth the original response of the platform.

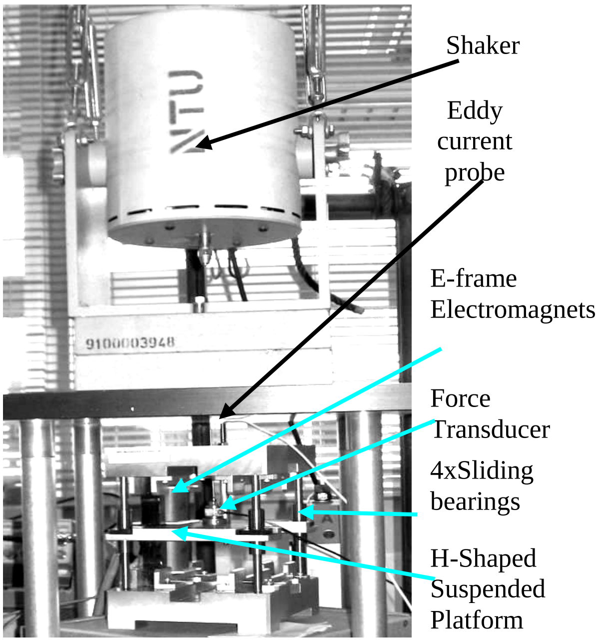

Figure 3: Photo of experimental setup

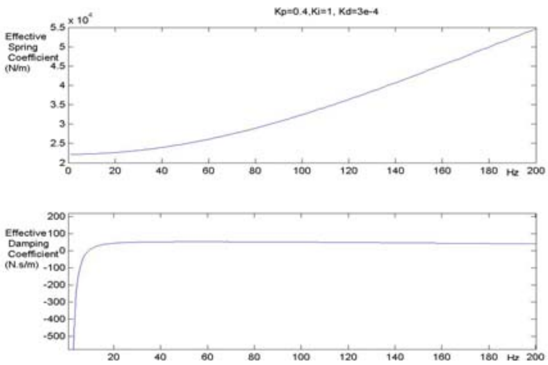

Figure 4: Effective stiffness and damping coefficients of electromagnets

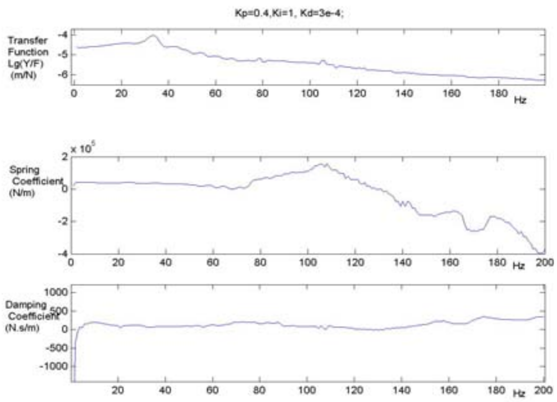

Figure 5: Actual stiffness and damping coefficients of electromagnets

Comparing Figure 4 and Figure 5, it can be seen that, actual damping coefficients are quite similar to effective ones through out the frequency range. The stiffness coefficients in frequency range of 1-107 Hz, actual stiffness coefficients are similar to effective coefficients, however, in the frequency range of 108-200 Hz, the effective stiffness is increasing, but the actual stiffness is decreasing. This is because effective characteristics of electromagnetic actuator are calculated in ideal conditions i.e. only requiring input and output of digital controller. However in actual experiments because of non-linearity in electromagnets, magnetic flux losses such as leakage, hysteresis, fringing effects, time-delay effects by eddy-currents and limited amplifier bandwidth [7], the actual characteristics will differ greatly from the effective ones. Therefore, effective characteristics of electromagnetic actuator cannot accurately represent its actual properties. To find out the exact relationship between the stiffness and damping of the electromagnetic actuator and PID controller gains, actual stiffness and damping coefficients should be adopted.

3.2. Influences of Controller Parameters on System Responses

By digitally varying the gain constants of PID controller, the actual stiffness and damping of electromagnetic actuator can also be changed. The system response that is represented as amplitude of close loop system transfer function can be used to illustrate the changes of system dynamic characteristics.

3.2.1. Influences of Varying Integral Feedback on System Responses

Figure 6 shows the effects of varying integral feedback on the system response. It can be seen from the system transfer function graph that increasing the integrator gain, the amplitude response of system at natural frequency will also be increased. It is also observed that integral feedback has dominant influences on low-frequency response but has negligible effect on the high-frequency response. This is consistent to integral feedback being well known to eliminate position error in the presence of static loads. Damping coefficients of electromagnetic actuator are decreased significantly with the increase in integrator gain, *Ki,* in low frequency range. Also, it can be seen that increasing integrator gain *Ki* attenuates the system dynamic responses in low-frequency range.

It needs to be mentioned that integrator gain has influences not only on damping of electromagnetic actuator, but also on its stiffness. Figure 6 shows that stiffness of electromagnetic actuator in low-frequency range is increased with the increase of *Ki*. From Figure 7, the expansion of Figure 6 around natural frequency, it can be seen that the actual stiffness coefficients are increased marginally with the increase in *Ki*, and therefore the natural frequency of system is increased, which is consistent with the fundamental of a second order dynamic system.

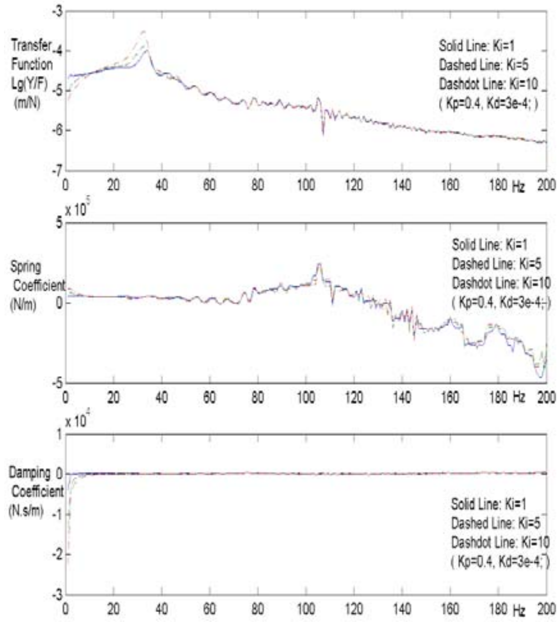

Figure 6: Characteristics of electromagnetic actuator with varying integral feedback

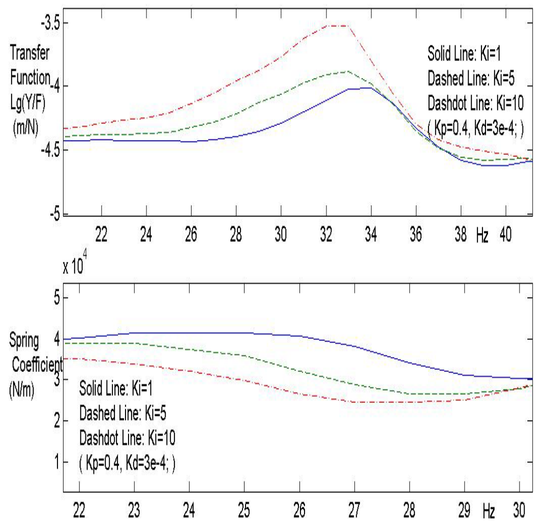

Figure 7: Expansion of Figure 6 around natural frequency

On the contrary, using their theoretical obtained *G(jω)* R.D. Williams, *et al*. [1] reported that integral term has little effect on the stiffness coefficients. This may be due to the effective stiffness coefficients calculations based on ideal conditions of equation (5). On the other hand, they observe the same phenomena that at resonant frequency rotor displacement amplitudes increased with the increase of *Ki*. And that system dynamic response in low-frequency range is attenuated by the integrator term.

3.2.2. Influences of Varying Derivative Feedback

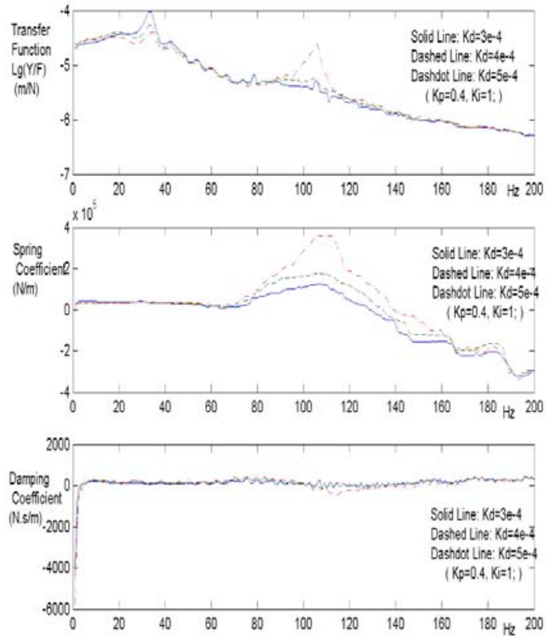

Figure 8: Characteristics of electromagnetic actuator with varying derivative feedback

From Figure 8 and Figure 9 it can be seen that increasing derivative gain can increase the damping coefficients, and therefore the amplitude of system response at the resonant frequency is reduced significantly. It is also noted in Figure 8 that stiffness coefficients in low-frequency range are affected little by the derivative gain, but in the high frequency range they are changed greatly by the derivative gain.

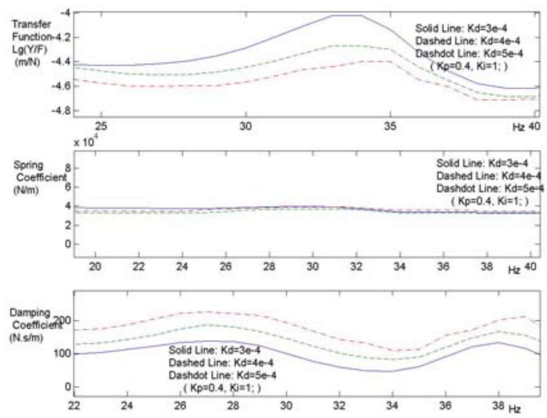

Figure 9: Expansion of Figure 8 around natural frequency

3.2.3. Influences of Varying Proportional Feedback

Figure 10 shows that by varying the proportional feedback gain has the expected effect of increasing system stiffness coefficients, and therefore the resonant frequency of system is increased. It needs to be mentioned that, increasing proportional gain also increases the system damping in low frequency range, and therefore system response amplitude before the resonant frequency is decreased, as it is shown in system response graph of Figure 10.

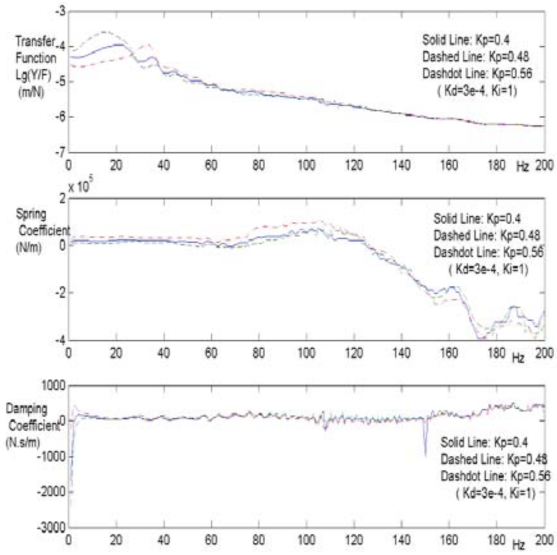

Figure 10: Characteristics of electromagnetic actuator with varying proportional feedback

4. Conclusions

It can be concluded from the above findings that proportional, integral, and derivative gains of PID controller have direct influences on both the actual stiffness and damping coefficients of a one-axis magnetically suspended system. That is to say, due to system time lags and losses, proportional gain not only changes the system stiffness but also its damping coefficients. This characteristic applies to both integral and derivative gains. The experimental setup for the single-axis magnetic suspension system also eliminates the cross-coupling capability between two axes in the AMB system. This is vital in providing fundamental understanding in an AMB system; prior to the complex system identification procedures. The need to model rotor stiffness coefficients and measurements of numerous linear, angular displacements and unbalance mass are also eliminated. The estimated stiffness and damping coefficients of the electromagnetic actuator will be useful for accurately predicting dynamic response of magnetically suspended system [10]. Toward this end, on going effort is directed towards developing this technique on our AMB system i.e. to extract both its actual and coupled supporting coefficients. With these estimated coefficients, we hope better predictions and optimal vibration control of rotor-bearing system.

5. References

[1] Ronald D. Williams, F. Joseph Keith, Paul E. Allaire, "Digital control of active magnetic bearings", *IEEE Transactions on Industrial Electronics*. Vol. 37, No. 1, pp 19-27, February (1990).

[2] Xie Z.Y., Xu L.X., Li Y., Qiu D.M., Yu L., "Influence of control parameters on dynamic characteristics of AMB system", *Journal of Aerospace Power*, Vol. 119, No.12, April (2004). (In Chinese)

[3] Humphris, R.R., Allaire, P.E., Lewis, D.W., Barrett, L.E., "Diagnostic and control features with magnetic bearings", *IEEE Proceedings of the 24th Intersociety 1989*, Vol. 3, pp 1491-1498, 6-11 Aug. (1989)

[4] Schroeder, M.R., "Synthesis of low-peak-factor signals and binary sequences with low auto correlation", *IEEE Transactions on Information and Theory*, pp 85-89 (1970).

[5] M.N. Sahinkaya, M.O.T. Cole, C.R.Burrows, "On the use of schroeder phased harmonic sequences in multi-frequency vibration control of flexible rotor/magnetic bearing systems", *Proceedings of the 8th International Symposium on Magnetic Bearings*, Mito, Japan, (2002)

[6] S.C. Mukhopadhyay. "Modeling of a Repulsive Type Magnetic Bearing for Five Axis Control Under Intermittent Operation Including Eddy Current Effect", *delta*, pp 425, *The First IEEE International Workshop on Electronic Design, Test and Applications* (DELTA '02), 2002

[7] G. Schweitzer, A. Traxler, and H. Bleuler, *Active Magnetic Bearings*, Verlag der Fachvereine (1994).

[8] Zhou, Y., Lim T.M., "On-line identification of squeeze-film dynamics of multi-mode rotor-bearing system", *Computer Assisted Mechanics and Engineering Sciences (Poland),* Vol. 5, pp 253 – 266 (1998).

[9] Burrows, C.R., Sahinkaya M.N., "Vibration control of multi-mode rotor-bearing systems," *Proc. Royal Society*, London, A386, pp 77-94 (1983).

[10] Chua, L.P., Song G.L., Yu, S.C.M. and Lim T.M., "Computational fluid dynamics of gap flow in a bio-centrifugal blood pump," Artificial Organ. Vol. 29, pp. 620-628 (2005).