

# Minimization Dynamic Error of the Sensors

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## Abstract

In spite of the advance technology the sensors still has systematic errors. The systematic errors are split into two parts: static and dynamics one. The purpose of this paper is to characterize a method of constructing algorithms for minimization dynamic error of the sensors

**Keywords:** Sensor, dynamic error, static error, minimized of the dynamic error

## 1 Introduction

In ideal case the value of measured environment on the sensors' input should be transform on the sensors' output with the same quality within the same time. The basic schema follows [1] is can see on the figure 1. But ideal cases still does not exist. The incongruity between measured environment presented as the input to sensor and sensor's output if the random errors are omitted, is due to systematic error. Systematic errors are usually divided into two parts, static error and dynamic error.

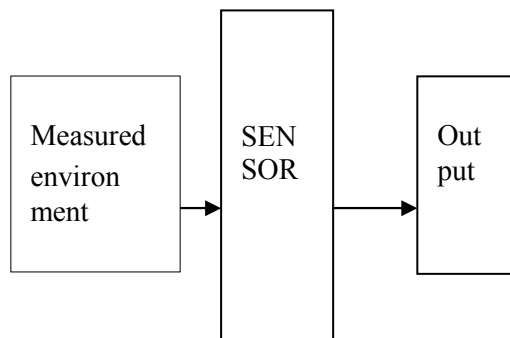


Fig. 1 General schema of the sensor function

Denoting the systematic errors of the output sensor by  $\Delta$ , they can be related to their corresponding true time signals  $y(t)$  as follows:

$$x_m(t) = y(t) \pm \Delta \quad (1)$$

where:  $\Delta = s + d$  (2)  
 $x_m(t)$  is measured environment on input of the sensor,  
 $y(t)$  is output of the sensor  
 $s$  - static error of the sensor;  
 $d$  - dynamic error of the sensor

Of course the influence of the dynamic error increases with increasing the velocity of moving measured environments.

## 2 Static error

In case of the static errors these are time independent. If we know sources of these kind errors we can minimize it by subtraction and/or addition of value this error to sensor's output and finally obtained errorless solution. The errors of the used methods for given measurement are included in the static error. Static error has main influence on the result of measurement when the sensor is in a steady environment after the transition event due to putting the sensor into the measurement environment.

## 3 Dynamic error

For the simplicity let suppose the first order differential equation for the model of sensor describing according equation (3)

$$a_1 \frac{dy(t)}{dt} + a_0 y(t) = x(t) \quad (3)$$

$$T \frac{dy(t)}{dt} + y(t) = x(t) = ct \quad (4)$$

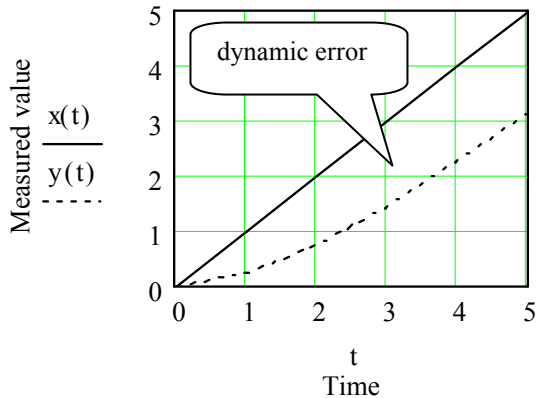


Fig. 2 Representation of the dynamic error

Where in equation (4) are:

$$x(t) = \frac{C}{a_0}t = ct \quad \text{- input of the sensor (moving environment)}$$

$c$  - velocity of the measured environment,  
 $t$  - time,

$$T = \frac{a_1}{a_0} \quad \text{- time constant}$$

Solving equation (4) give equation (5), where from error point of view the part (6) yields dynamic error.

$$x(t) = y(t) + cT \left( 1 - \exp\left(\frac{-t}{T}\right) \right) \quad (5)$$

$$T \frac{dy(t)}{dt} = cT \left( 1 - \exp\left(\frac{-t}{T}\right) \right) \quad (6)$$

#### 4 Minimization dynamic error

One way for minimization dynamic error, describing in chapter above is put equation (6) zero. This way is not as easy as one view of the first look. If is supposed digitalized output signal from sensor as impulse series, one can derived the errorless signal from the three samples with equidistant time distance. The samples are in the fig. 3.

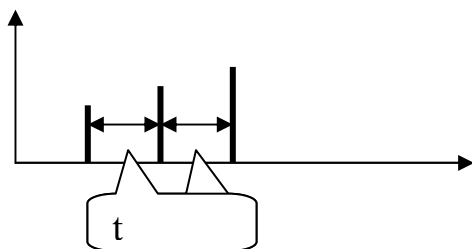


Fig. 3 Three samples of the measured environment on the sensor output

From the picture 2 is clear, that the dynamic error within the time  $t_i$  is the same as dynamic error within the time  $t_{i+1}$ , of course after finishing transitional event, supposing the velocity  $c$  of measured environment is equal constant. Now from (4) for the two times  $t_i$  and  $t_{i+1}$  respectively, supposing  $c = \text{const.}$  we obtain following expression of time constant  $T$  (7).

$$T = \frac{t_i y(t_{i+1}) - t_{i+1} y(t_i)}{t_{i+1} \frac{dy(t_i)}{dt} - t_i \frac{dy(t_{i+1})}{dt}} \quad (7)$$

For the further derivation we need introduce auxiliary time variable much close to central sample than sample  $t_i$  and  $t_3$ . The role of these auxiliary time variable let play  $t_i$  and  $t_{i+1}$ . This situation is on the fig. 4, where the times  $t_i$  and  $t_{i+1}$  are as follows:

$$t_i = \frac{t + 2t}{2} \quad t_{i+1} = \frac{2t + 3t}{2} \quad (8)$$

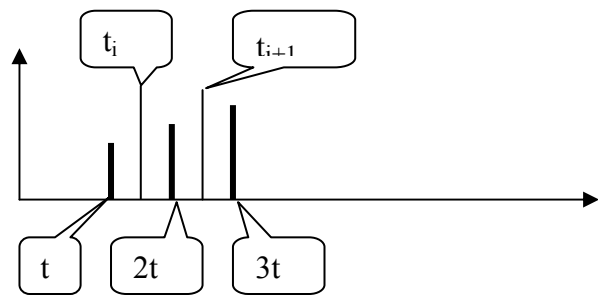


Fig. 4 denoting of the three samples

The auxiliary variables:

$$\left. \begin{aligned} \frac{dy(t_2)}{dt} &\cong \frac{y(3t) - y(t)}{3t - t} \\ y(t_i) &= \frac{y(2t) + y(t)}{2} \\ y(t_{i+1}) &= \frac{y(3t) + y(2t)}{2} \\ \frac{dy(t_{i+1})}{dt} &\cong \frac{y(3t) - y(2t)}{3t - 2t} \\ \frac{d(t_i)}{dt} &\cong \frac{y(2t) - y(t)}{2t - t} \end{aligned} \right\} \quad (9)$$

The final equation for minimizing dynamic error with taking into account three samples one can obtain by

putting the expressions (9) and (8) to (7) and finally to (4).

After some manipulation the final equation for minimizing dynamic error with taking into account three samples yields to expression (10), where  $Y(t)$  is corrected output signal from the sensor and  $y(t)$  is output signal from the sensor.

$$Y(t) = y(t) + \frac{3 \cdot (y(3 \cdot t))^2 - 2 \cdot y(2 \cdot t) \cdot y(3 \cdot t) - 8 \cdot y(3 \cdot t) \cdot y(t) + 2 \cdot y(2 \cdot t) \cdot y(t) + 5 \cdot (y(t))^2}{4(8 \cdot y(2 \cdot t) - 5 \cdot y(t) - 3 \cdot y(3 \cdot t))} \quad (10)$$

Table 1. The results of the experiments with relative error expressing

Measured moving environmet  $x(t)$	Original sensor response  $y(t)$	Original sensor response error  $\frac{x(t) - y(t)}{x(t)} 100$ [%]	Corrected sensor response  $Y(t)$	Corrected sensor response error  $\frac{x(t) - Y(t)}{x(t)} 100$ [%]
0.000	0.000	0.000	0.000	0.000
1.000	0.368	63.212	1.152	-15.173
2.000	1.135	43.233	2.055	-2.757
3.000	2.050	31.674	3.016	-0.520
4.000	3.018	24.542	4.004	-0.099
5.000	4.007	19.865	5.001	-0.019
6.000	5.002	16.625	6.000	-3.295e-3
7.000	6.001	14.273	7.000	-4.495e-4
8.000	7.000	12.496	8.000	1.736e-6
9.000	8.000	11.110	9.000	3.829e-5
10.000	9.000	10.00	10.00	2.272e-5

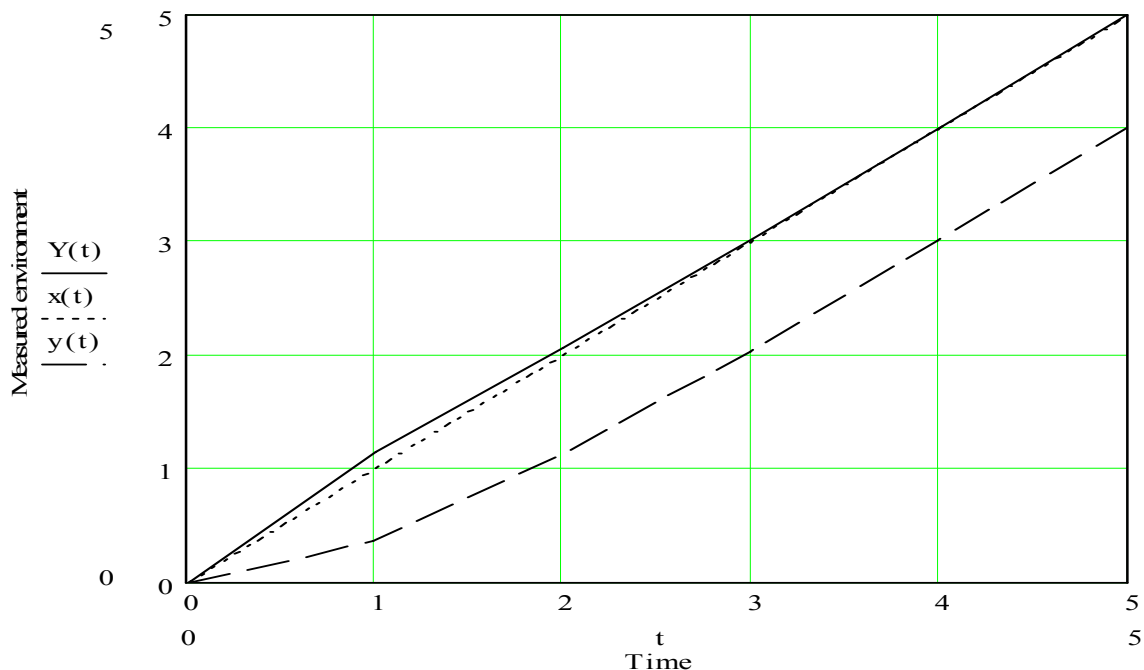


Fig. 5 Graphic way of represents the results of equations (4), (10) and expression  $x(t)$ .

## 5 Results

The results of the experiments are in detail in the table 1, where is compared no corrected signal from sensor with corrected signal from sensor and for both is calculated relative error. From the table 1 one can see that after some steps the dynamic error rapidly decreasing and surprisingly we obtain the same signal on the output of sensor as the moving environment on the input of the sensor. Within the table 1 the time constant  $T$  and velocity of the moving environment  $c$  was put equal 1. Of course if one changes these constants, the results are changing. The graphic way of presents results are in the picture 5.

## 6 Conclusion

The one way of the minimizing of the dynamic error of the sensor was shown. As the simple example was the first order differential equation was supposed, as the sensor model. Of course the expression (10) is possible to use for higher order of the sensor's model, but the additional errors due to difference of the sensor's model order was exist. This problem will be discussed and shown during presentation this contribution.

Finally is possible conclude that suggested solution give good results. This solution do not use the transfer function as [2], but improving some expression published in [3] and for practice use is suitable, because work only with output's samples of

sensor signal. Of course, one can with varying the time constant  $T$  and velocity of the measured environment  $c$ , obtain plenty of pictures and tables, but these are not important for representation the idea of minimizing dynamic error describing by this method.

## 7 References

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