1st International Conference on Sensing Technology
November 21-23, 2005 Palmerston North, New Zealand

A Data-Fitting Approach for Displacements and Vibration Measurement using Self-mixing Interferometers

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Abstract

This paper presents a signal processing approach for vibration measurement using self-mixing interferometer (SMI). Compared to existing approaches, the proposed approach is able to achieve an accuracy of $\lambda/40$ which significantly exceeds the accuracy limit associated with conventional simple SMI systems $\lambda/4$.

Keywords Optical feedback, self-mixing interferometer, data-fitting

1 Introduction

Optical feedback self-mixing interferometer (OFSMI) is an opto-electronic system that makes use of the self-mixing interferometric effect of semiconductor lasers (SLs). When a small potion of the light emitted by a laser diode (LD) is reflected or back scattered by an external object and re-enter the laser cavity [1]. The reflected light will be interfered with the light inside the cavity which causing the variance of the overall laser intensity emitted by the LD. During the past decade this physical phenomenon has been studied extensively and many possible applications have been investigated. A major application is the measurement of metrological quantities associated with the external object, including the displacement, vibration parameters, and speed and so on [2,3,4,5,]. A well known conclusion regarding the OFSMI based measurement is that the accuracy or resolution is limited by $\lambda/4$ with the simple OFSMI setup with constant driving current for the LD and without adding other extra optic components. In order to achieve higher accuracy beyond the limitation, some proposed methods for analysis of SMI signal have been used, such as calculating the phase radio based on pseudo-heterodyne method [6], double external cavity method combined with FFT technique [7], phase-locked technique [8].

Recently, a data-fitting technique has been proposed for the measurement of the Linewidth Enhancement Factor (LEF) of semiconductor lasers [9], using OPSMI systems, which shows a significant improvement in terms of accuracy and resolution as compared to other conventional techniques. In this paper, we try to use a similar data-fitting approach to measure vibration parameters, the amplitude in particular.

2 Theory

The research to the OFSMI has led to a well known mathematical model as follows [10][11]:

$$\phi_F(L) = \phi_0(L) - C \cdot \sin[\phi_F(L) + \arctan(\alpha)] \quad (1)$$

$$P(L) = P_0[1 + mG(L)] \quad (2)$$

$$G(L) = \cos(\phi_F(L)) \quad (3)$$

Where L is the length of the external cavity. Equation (1) is called the phase condition. In this equation, α is the LEF, and the parameter $\phi_0(L)=4\pi L/\lambda_0=\phi_0(\tau)=\omega_0\tau$, is the phase in the external cavity without optical feedback effect, while $\phi_F(L) = \phi_F(\tau) = \omega_F(\tau)\tau$ is derived by the actual change of the phase with the effective feedback C. From the experiment, $\phi_F(L)$ will vary where there is a change of the external cavity length.

Equation (2) provides the relationship with the intensities of the laser emitted by the SL with or without the external feedback effect. It is seen that with the external cavity, the laser output power deviate from P_0 by a factor of $mG(\phi_F(L))$ where m is called modulation index (typically $m = 10^{-3}$), and $G(\phi_F(L))$ is called the interferential function which gives the effect of the external cavity length.

According to these three equations, the location of the external object can be obtained by analyzing the phase function with feedback.

Consider the case that the external target is subject to a harmonic vibration, that is:

$$L(t) = L_0 + \Delta L \sin(2\pi ft + \theta_0) \quad (4)$$

So, $\phi_0(L)$ is given by:

$$\phi_0(L(t)) = A_0 + A_1 \sin(2\pi f t + \theta_0) \quad (5)$$

where $A_0 = {4\pi L_0}/{\lambda_0}$ and $A_1 = {4\pi\Delta L}/{\lambda_0}$.

The SMS signal $P(L)$ is acquired by an A/D card with the sampling frequency *fs*. Thus, we can get the following discrete model through the sampling algorithm:

$$\phi_0(n) = A_0 + A_1 \sin(2\pi({f_s}/{f})n + \theta_0) \quad (6)$$

$$\phi_F(n) = \phi_0(n) - C \cdot \sin[\phi_F(n) + \arctan(\alpha)] \quad (7)$$

$$G(n) = \cos(\phi_F(n)) \quad (8)$$

Note that for the sake of simplicity we used $\phi_F(n) = \phi_F(L(n))$ and $G(n) = G(L(n))$, where n is the discrete time index. The task of vibration measurement is to determine the values of the frequency f and the amplitude A_1.

3 Parameter Estimation Technique

As we only get the SMI data *P(n)* from the experiment, we should pre-processed the data to obtain *G(n)*. This process includes removal of P_0 and additive noise, normalization of the amplitude. From Equations (6)-(8) we see that Φ_0, Φ_F, f, α and C are all unknown variables and need to be determined.

Firstly the vibration frequency f can be determined by the auto-correlation function of given by:

$$r_G(m) = \frac{1}{N} \sum_{n=0}^{N-1-m} G_N(n) \cdot G_N(n+m) \quad (9)$$

Where N is number of samples of $G(n)$ used for calculating the auto-correlation function. m is time delay index which varies from *–(N-1)/2* to *(N-1)/2*. Due to the simple harmonic vibration of the object, $G(n)$ will also be periodic with the same fundamental period given by $N_0 = f_s / f$. It is well known that auto-correlation functions of periodic signals exhibit peaks at the integer multiples of their fundamental period. By detecting the location of the peaks in $r_G(m)$, N_0 (and thus f) can be obtained.

3.1 Estimation of Displacement

Now we consider the issue of determining A_0 and A_1. From Equations (6), (7) and (8) we see that due to the cosine function in (8) we are only able to recover up to a range of $(0, \pi)$ for A_0 as any value of $2m\pi + A_0$ will yield the same $G(n)$ .Hence we assume that $A_0 \subset (0, 2\pi)$. A_1 is the vibration amplitude, which can separated two parts $A_2 + A_3$, where A_2 is the integral multiple of $m\pi$, that can be determined from the $G(n)$ directly by fringe counting. Hence we simply need to estimate $A_3 \subset (-\pi/2, \pi/2)$ in order to achieve higher accuracy over the conventional techniques.

To determine A_2 and A_3, we employ data-to-theoretical model fitting techniques. The idea is that for given observed SMI data segment, we find out the parameter values with which the theoretical model yields a SMI signal waveform that is closest to the observed SMI data. For this purpose, we define the following cost function [9]:

$$F(\hat{A}_0, \hat{A}_1) = \sum_{n=1}^{N} \left\{ G(n) - \hat{G}(\hat{A}_0, \hat{A}_3, A_2, f, k, C) \right\}^2 \quad (10)$$

Where N is data length of $G(n)$ used for estimating the parameters. $\hat{G}(\hat{A}_0, \hat{A}_3, A_2, f, k, C)$ is the value based on computation using models (6-8) incorporating the estimated values of $\hat{A}_0$ and $\hat{A}_3$. It is expected that minimization of the above defined cost function with respect to the parameters will yield the estimations of these parameter values.

However, there is one more issue that needs to be studied before we can use the above cost function. In most cases the values of C and α are also not known, and their measurement is a challenging issue as well. Hence we should study the influence of their values to the cost function as follows.

Firstly, we generate a set of SMI signal using the model in (6), (7) and (8) with the parameters $(C = 0.5, \alpha = 6, A_0 = 20\pi/30, A_2 = 5\pi, A_3 = 10\pi/40)$, the SMI waveform is shown in Figure 1 :

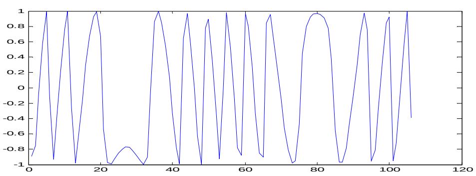
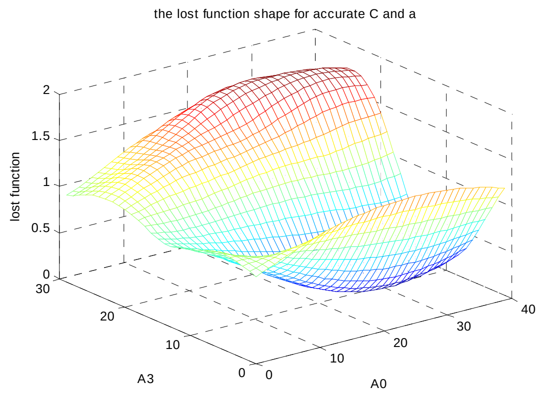
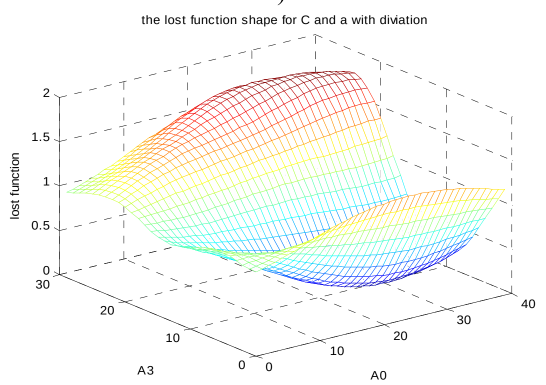


Figure 1 Generated G(n) .

Secondly, with the generated SMI data, we look at the influence of A_0 and A_3 on the minimum of the cost function. Figure 2 (a) shows the shape of the cost function with respect to A_0 and A_3, while C and α take the true values. We also studied case where C and α are not the true values and found that the cost function has a global minimum at the location where A_0 and A_3 take the true value. As an example, Figure 2 (b) gives the shape of the cost function when C and α take the values of 0.2 and 4 respectively which are not the true values. As a result, we can say that the cost function can be employed to estimate A_0 and A_3 even the value of C, α are known as they almost do not affect the optimal location of the cost functions.



a)



b)

Figure 2: The shape of the cost function for different C and α. Note that $A_0 \subset (0,2\pi)$ and $A_3 \subset (-\pi/2,\pi/2)$ with 40 and 30 increments respectively.

4 Gradient-Based Optimization Algorithm

The next issue to be considered is to estimate the A_0 and A_3 by minimizing the cost function. As there is only one global minimum with the cost function, we use the gradient-based algorithm as follows:

$$\hat{A}_{0\,j} = \hat{A}_{0\,j-1} - \mu_1 \frac{\partial F}{\partial \hat{A}_0}\bigg|_{\hat{A}_0=\hat{A}_{0\,j-1}} \tag{11}$$

$$\hat{A}_{1\,j} = \hat{A}_{1\,j-1} - \mu_2 \frac{\partial F}{\partial \hat{A}_1}\bigg|_{\hat{A}_1=\hat{A}_{1\,j-1}} \tag{12}$$

$$\frac{\partial F}{\partial \hat{A}_0} = 2\sum_{n=1}^{N}\left\{G(n) - \hat{G}(f,k,C,A_2,\hat{A}_0,\hat{A}_1)\right\} \times \sin(\hat{\phi}_F(n))/(1+C\cdot\cos(\hat{\phi}_F(n)+k)) \tag{13}$$

$$\frac{\partial F}{\partial \hat{A}_3} = 2\sum_{n=1}^{N}\left\{G(n) - \hat{G}(f,k,C,A_2,\hat{A}_0,\hat{A}_3)\right\} \times \sin(\hat{\phi}_F(n)) \times \sin(2\pi f n)/(1+C\cdot\cos(\hat{\phi}_F(n)+k)) \tag{14}$$

Where μ_1 and μ_1 are the step sizes.

Computer simulations have been performed to test the above algorithms and the results are shown in Table 1. For each case, A_0, A_3 have been computed by gradient algorithm for 15 times and the average is taken as the final estimation. It is seen that we can always obtain very accurate estimation of A_0, A_3.

5 Experiment Setup and the Results

The SMOFI experimental setup is shown in Figure 3. The SL is biased with a dc current and a lens is used to focus the light on a metal plate which is considered as an external target. This target is made to vibrate harmonically by placing it closed to a loudspeaker which is driven by a sinusoidal signal. The SMS is detected by the monitor photodiode and is amplified by a trans-impedance amplifier. Then the amplified signal is obtained by personal computer by an A/D card at the sampling frequency of 20 kHz.

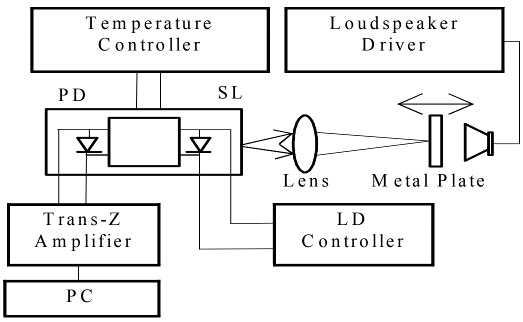


Figure 3. The OFISM experimental set-up used for obtaining SMSs

The SMI signal obtained is shown in Figure 4.

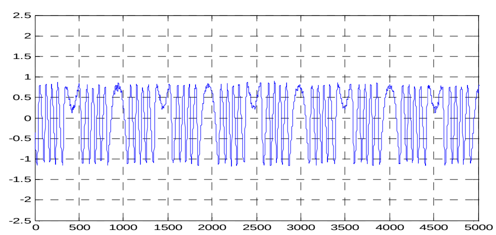


Figure 4. The SMI signal obtained by photodiode in experiment

Using the fitting principle, we obtained the displacement according to the SMS waveform By applying the gradient-based algorithm we can get the estimated values of A_0 (L_1/λ) and A_3 ($\triangle L/\lambda$, the integral multiple of $m\pi\lambda$ has been ignored.) which are 0.2167 and 0.0500 respectively. Note that due to the limitation of our experimental facility we are not able to verify the results with other means of measurement. However from the simulations in Section 5 we can say that the measurement error can be up to 5%.

Table 1: A_0, A_3 simulated by Gradient Algorithm in different C and α.

C	α	A'_0	A'_3	$A_0(sim)$	$A_3(sim)$	$Error(A_{0sim})$	$Error(A_{3sim})$
0.2	3	4π/6 (2.0994)	10π/40 (0.7800)	2.1332	0.7407	1.85%	4.12%
0.2	7	4π/6 (2.0994)	11π/40 (0.8530)	2.0686	0.7739	1.22%	1.64%
0.4	3	44π/60 (2.3038)	11π/40 (0.8530)	2.1278	0.7629	1.59%	2.86%
0.4	7	44π/60 (2.3038)	11π/40 (0.8530)	2.1226	0.7983	3.03%	1.64%
0.6	3	4π/6 (2.0994)	10π/40 (0.7800)	2.0882	0.7303	1.03%	2.4%
0.6	7	4π/6 (2.0994)	10π/40 (0.7800)	2.0783	0.8234	3.00%	4.84%
0.8	3	4π/6 (2.0994)	10π/40 (0.7800)	2.1054	0.8404	0.94%	7%
0.8	7	44π/60 (2.3038)	8π/40 (0.6380)	2.0750	0.7657	9.30%	1.77%
0.5	6	4π/6 (2.0994)	10π/40 (0.7800)	2.0621	0.7831	1.54%	0.29%

Note: $C=0.5$, $\alpha=6$ are the values which generate $G(n)$, and $A_0=4\pi/6$ and $A_3=10\pi/40$ are the accurate parameters for $G(n)$ in the cost function. The location for A'_0 and A'_3 is the lowest point for the cost function in Part 3.1, $A_0(sim)$ and $A_3(sim)$ is the value computed by Gradient algorithm with SA, $Error(A_{0sim})$ and $Error(A_{3sim})$ are the error for the value generated by Gradient Algorithm.

6 Conclusion

This paper has presented a new approach to measure displacement using OFSMI in weak optical feedback region. The validity of fitting principle and SA algorithm can be confirmed by the compute simulation and experiments. Compared to the method proposed by Ming. Wang [7], [12], the proposed approach achieves similar accuracy but is much simple in implementation.

7 References

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