A ‘ZMP’ management scheme for trajectory control of biped robots using a three mass model

Jagannathan Kanniah* (jagkan@sp.edu.sg), Zar Ni Lwin*, D Prasanna Kumar*, and Ng Nai Fatt

*Singapore Polytechnic
500 Dover Road
Singapore 139651

Abstract:
Development of simple computational schemes to manage zmp to force biped robots into dynamic walking has been an important area. In this paper, a straightforward computational technique to map the body trajectories of a biped, to achieve dynamic walking is suggested. While such works for one mass representation has been reported earlier, here, a three mass representation of a biped robot is considered. Using the swinging leg mass as the prescribed motion, compensatory motion of the hip is computed. Easily manageable and computationally viable technique to implement such a scheme is described. Such technique is applied to an example and the body trajectory and the resulting ‘zmp’ are presented.

1. Introduction
Biped research has been an important area of robotics for the past two decades [1]. Among many techniques worked on, the important concept is trajectory control of joint angles. The other technique widely discussed in literature is the force control of joints [2-8]. The users of trajectory control method have heavily relied upon playback techniques and this has been successfully demonstrated by many researchers. It is well known that the robot does not fall, even though the centre of mass (COM) of the robot may be outside the perimeter of support for a certain duration, if some Zero Moment Point (ZMP) conditions are met. An analytical technique to compute the body trajectories to achieve the above has been elusive. Many approximations are necessary. Some attempts have been reported recently.[9-13] Heuristic body trajectory computation also has been attempted.[14-15] ZMP management for biped with mass-less legs has been reported recently.[16,17] Such techniques make some restrictive assumptions. This paper aims at providing a reasonably straightforward

 computational technique for mapping body trajectory to achieve desirable zmp management for three masses representation of a biped robot. Several techniques for calculating and measuring, and controlling ZMP has been reported [9-13]. Most such techniques are not strictly applicable for dynamic walking trajectory controlled robot.

The sequence of biped walking is shown in Fig.1.

![Figure 1: Requirement of ZMP and COM movement while biped walks](image)

With reference to Fig.1, the analysis starts when one swing leg-1 lands ahead of the hip-mass at time \( t = t_s \) and becomes the new stance leg. Since stance leg is the only support the ZMP should fall within the perimeter of the stance-leg’s foot area. Just after this instant the other legs behind (2) lifts off and starts swinging forward to span double the step size, \( 2X_s \), while the hip COM moves to span the step size. Until this time, \( t = t_s + T \), the ZMP should stay with in the stance-leg’s foot, A. This
process continues as the current stance leg again becomes the swing leg. Obviously the ZMP should now shift swiftly to point B. The sequence continues with legs swinging and supporting alternatively.

2. Solution for single mass case [16,17]

A detailed analysis of such single mass system has been provided elsewhere while considering only the sagittal plane. The constraint on hip mass movement can be expressed as

\[ z = k_x x + k_y y + z_c \]

where \( z_c \) is the height of hip, \( x, y \), and \( z \) are appropriate co-ordinates. To simplify matters, one may assume that hip moves on a horizontal plane. This is accomplished by setting both \( k_x \) and \( k_y \) to zero. Then, the similarity with the inverted pendulum will not be valid anymore, since in order to satisfy Eqn.1, the limb has to change its length. This is quite acceptable for two legged robots with knees.

From the basic definition of ‘ZMP’ [1] and considering only sagittal plane, it can be written,

\[ p_x = x - \frac{z_c}{g} \]

where \( p_x \) is the stipulated ZMP. The above Eqn.2 provides a way of computing the ZMP in a single foot phase, which can be likened to an inverted pendulum. Above equation is intuitively appealing. Then, Eqn.2 can be used as a servo system and trajectories can be computed, while \( p_x \) is the input. The solution has been obtained by using LQC technique [16]. But authors have provided an alternative solution, using simple analytical techniques [17] as

\[ x_{com} = p_x + \left( \frac{X_0}{2} - \frac{p_x}{2} - \frac{V_0}{2\alpha} \right) e^{\alpha t} + \left( \frac{X_0}{2} - \frac{p_x}{2} + \frac{V_0}{2\alpha} \right) e^{-\alpha t} \quad 0 \leq t \leq T_s \]

where, \( X_0 \) is the initial position of mass, when the next swing starts, \( V_0 \) is the initial velocity of the hip mass, \( T_s \) is the step time and \( \alpha = \frac{2}{T_s} \).

First Stage:
The duration in Fig.1, time duration between the instant when leg 2 starts the swing and lands at B, is called the stage one. During this stage, the ZMP needs to be under the stance foot at point A. The step size used is \( X_s \). Then equating \( p_x=0 \), \( X_0= -\frac{X_s}{2} \) and assuming that \( V_0 =0.7 \text{m/s} \), appropriately, the results are plotted in Fig.2, for 600 points, using a sampling time of 1 ms. Note that starting and ending hip velocities have to be same for continuous walking. In Fig.1 it should be pointed out that \( t_i \) is not necessarily half of step time \( T_s \). Actually it is the instant at which COM coincides with ZMP.

Second stage:
Second stage begins when the leg-2 becomes the stance leg placed at B and the leg-1 starts the swing and the zmp is to be swiftly moved to B. Since there would be continuity of initial and final conditions of each stage, the dynamics are quite similar. The actual \( X_{com} \) trajectory is shown in Fig.2 starting from point 601 to 1200.

3. Three mass system analysis

The single mass system described in section 2 provides a way of solving the ZMP management problem for an unrealistically simple case, since the biped is a system of many masses. It is obvious such simplistic model based solution may not be applicable for a real biped system. Analytical solutions are inappropriate for real time application. However, numerical computation techniques may be used for solving such systems, if the calculations are manageable in real time. In this section, we attempt to solve a three mass system, with the heaviest mass placed on the hip and the two other masses being at the two feet. Such an assumption is reasonable in many robots, since the driving motors for the knees are closer to the upper part of thigh and the ankle driving motors are closer to the feet. Furthermore, in this work the assumption regarding the hip movement on a horizontal plane is also abandoned. Constraint in Eqn.1 is not applicable anymore.
Instead it is assumed that the stance leg is kept straight while the swing leg gets folded at the knee. Such a situation is depicted in Fig.3.

The starting point of the discussion is when the left leg (thick line) is stance leg and the right leg (thin line) is swinging. During this period, the ZMP is under the left foot. The ZMP continues to stay there until the right foot lands. At this time, the ZMP should switch to the right foot when the left becomes the swing leg and the ZMP stays under the right foot until the left foot lands. To make things manageable we confine our discussion to the situation shown in Fig.4.

Figure 3 : The walking process( Three mass biped)

Figure 4: Hip and swing leg motion( Three mass biped)

The inverted pendulum movement of stance leg is depicted in Fig.4. The stance leg is not folded as in realistic situations. The angle of inclination of the stance leg is assumed to go from negative to positive. Let it be \( \theta \) and the angular velocity be \( \omega \), which varies with respect to time to accommodate ZMP requirements.

The tangential velocity of the hip at any time is \( v \), the velocity in x-direction is \( v_{x2} \), and the velocity in z-direction is \( v_{z2} \).

Then,

\[
\begin{align*}
v &= \omega \\
v_{x2} &= v \sin \theta \\
v_{z2} &= v \cos \theta
\end{align*}
\]

The position of the hip mass in x and z directions can be written as,

\[
\begin{align*}
x_{z} &= l \sin \theta \\
z_{z} &= l \cos \theta
\end{align*}
\]

The acceleration of the hip mass in x and z directions can be written as,

\[
\begin{align*}
\dot{x}_{z} &= -l \omega^2 \sin \theta + l \cos \theta \omega \\
\ddot{z}_{z} &= l \omega^2 \cos \theta + l \sin \theta \omega
\end{align*}
\]
In this paper, we assume that the swing foot motion is the prescribed motion and hip performs the compensating motion. The horizontal motion of the swing foot is denoted by $x_3$. Since it is prescribed, one can define an arbitrary, but reasonable path. Here the prescribed motion, $x_3$, is assumed to be,

$$V_s = 2 \frac{X_s}{t_s}$$

$$x_3 = V_s t - X_s$$

$$\dot{x}_3 = V_s$$

$$\ddot{x}_3 = 0$$

Note that the swing size is double the step size. Furthermore, assuming the foot-lift to be $g_s$, the foot clearance, $z_3$, can be prescribed to be,

$$z_3 = g_s \sin(\frac{\pi}{t_s} t)$$

$$\dot{z}_3 = g_s \frac{\pi}{t_s} \cos(\frac{\pi}{t_s} t)$$

$$\ddot{z}_3 = -g_s \left(\frac{\pi}{t_s}\right)^2 \sin(\frac{\pi}{t_s} t)$$

Taking the moments about the stance leg foot, one can write,

$$m_2 (\ddot{z}_2 + g) x_2 - m_2 \ddot{x}_2 z_2 + m_3 (\ddot{z}_3 + g) x_3 - m_3 \ddot{x}_3 z_3 = 0$$

Substituting for $\ddot{x}_2$, $\ddot{z}_2$, and solving for angular acceleration, we obtain

$$\omega = \left(\frac{m_2 (x_2 g + x_2 \omega^2 \cos \theta) - m_3 (x_3 g + x_3 \omega^2 \cos \theta) + m_3 (\ddot{z}_3 + g) x_3 - m_3 \ddot{x}_3 z_3}{m_2 (z_2 \cos \theta - x_2 \sin \theta)}\right)$$

As a reverse process the ZMP also can be computed for verification using,

$$X_{\text{ZMP}} = \left(\frac{m_2 (\ddot{z}_2 + g) x_2 - m_2 \ddot{x}_2 z_2 + m_3 (\ddot{z}_3 + g) x_3 - m_3 \ddot{x}_3 z_3}{m_2 (\ddot{z}_2 + g) + m_3 (\ddot{z}_3 + g)}\right)$$

**Single mass case:**
The above case can be converted to single mass system simply by setting $m_3$ to zero. It is done for comparison with one mass system. The resulting COM trajectory is shown in Fig.5. The reader is urged to compare this with Fig.2. Both Fig.2 and Fig.5 show two full steps. The initial velocities were adjusted to be same. Slight differences in velocity changes are noticeable. This is acceptable since in the first result, the hip moves horizontally while in the next result the hip moves on an arc. The ZMP for both cases are plotted in meters. One can see the similarity in the response, which serves as a verification of the computational technique. In Fig.5, ‘A’ is an intermediate variable, used for debugging.

![Figure 5: Iterative trajectories solution(Single mass biped-hip moving on an arc)](image)

**Three mass case:**
When the feet masses are included, the COM trajectories can be computed using the same set of equation and the results are shown in Fig.6.

By comparing Fig.5 and Fig.6 one can see that the stance leg trajectory angles have different shapes. This is the effect of the mass $m_2$ suffering the prescribed motion and the hip actually compensating it. The ZMP jump is quite sharp, which will guarantee dynamic walking possible. In this technique, one can include additional masses by appropriately including an inverse kinematics engine in the equations. The angles are reset as the swing leg lands. The hip mass movement and angular velocities $s$ are continuous. In comparison with Fig.5, where there is no foot mass, Fig.6 shows higher changes in velocity and vertical movements of the hip.

4. **Implementation**
Such a system of control is being implemented on a biped. The system uses an eZdsp™ mother board featuring TMS320LF2407.
processor as the master, and a set of five Intel®
80C196KC based system boards as slaves. Four
out of the 5 slaves are used for implementing the
command angles received from the eZdsp
master board.

Figure 6 : Hip movement in three mass biped system

The fifth slave is for sensor information, which
is sent to the master at constant intervals. Every
80C196KC processor is in charge of DDC
direct digital control) implementation to control
the joint angles. Every joint motor is controlled
using servo control loop. Three processors
handle three motors each and the fourth one
handles only one motor. The serial
communication takes place at a speed of
100Kbauds, which is pretty comfortable. The
test results will be reported elsewhere.

5. Conclusion
An alternate scheme to implement ZMP
management for a trajectory controlled biped
robot has been presented. It is shown that using
simple calculations, the COM reference(s) can
be generated. Such references are easy to
implement using joint angle servo controllers
using appropriate techniques. This method is
being implemented in an actual biped robot and
results are encouraging.

Further work is underway to build and
program a full humanoid robot, using a
complete CAN (control area network) for inter-
processor communication and additional
instrumentation.

Figure 7 : Photograph of the biped
6. References


