Design and Analysis of a Four-wheel Omnidirectional Mobile Robot

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Abstract

This paper addresses the issue of the development of a four wheel omnidirectional mobile robot. The modeling and control of the robot system are discussed and some experiment results are presented.

Keywords: Omnidirectional Mobile Robots, Time Constant, Real Time Control

1 Introduction

Omnidirectional wheeled mobile robots are superior to those with differential wheels in terms of dexterity and driving ability [1][2][3]. They are able to perform various movements difficult or impossible for differential wheeled mobile robots.

Among omnidirectional mobile robots, the one with four or more wheels is more powerful than that with three wheels. In this paper, the design of the robot and its modeling are presented. The robot's motion control system and its real time implementation are described.

The paper is organized as follows. In Section 2, the kinematic model of an OWMR is analyzed. In Section 3, the controller design and experiment study are described. The conclusion is given in Section 4.

2 Kinematic model

Figure 1 shows a top view of an OMWR robot . The coordinate $X_r O_r Y_r$ is attached to the robot body and XOY denotes the world coordinate. The radius of the wheel and the radius of the robot body are denoted by r and b respectively. The angle between the axles of the wheels are denoted by α_i $(i = 1, 2, 3, 4)$ respectively. The angular velocity of each wheel is denoted by ω_i $(i = 1, 2, 3, 4)$ and the direction of the linear velocity of the center of the wheel is indicated by v_i $(i = 1, 2, 3, 4)$ with respect to the coordinate $X_r O_r Y_r$. The angle between the coordinates $X_r O_r Y_r$ and XOY is denoted by θ. The linear and angular velocities of the robot are denoted by $v = [v_x \ v_y]^T$ and ω respectively. The overall dimensions of the physical robot are: 180mm (Diameter) X 110mm (Height). The mechanical drawing and the photo of the robot can be found in Figures 2 and 3.

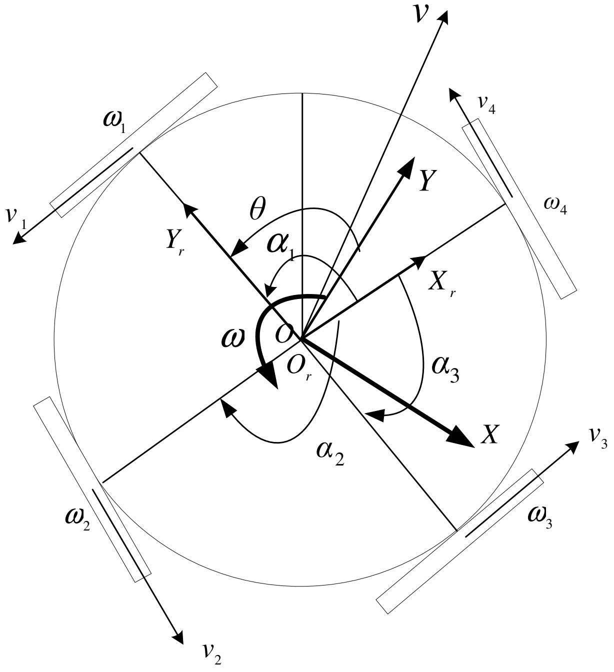

Figure 1: Omni-wheeled Robots

In our design, each wheel keeps the same distance (b) from the center of the robot. Assume the axis x_r align with the axile of the 4th wheel. The following kinematic relation exists,

$$r\omega_1 = b\omega + v_r^T v_1 \tag{1}$$
$$r\omega_2 = b\omega + v_r^T v_2 \tag{2}$$
$$r\omega_3 = b\omega + v_r^T v_3 \tag{3}$$
$$r\omega_4 = b\omega + v_r^T v_4 \tag{4}$$

where

$$\begin{aligned} v_1 &= [-\sin(\alpha_1) \ \cos(\alpha_1)]^T \\ v_2 &= [-\sin(\alpha_2) \ \cos(\alpha_2)]^T \end{aligned}$$

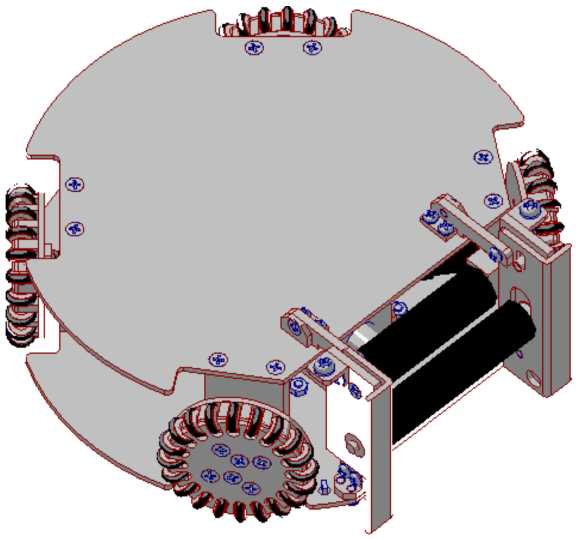

Figure 2: Mechanical drawing of the robot

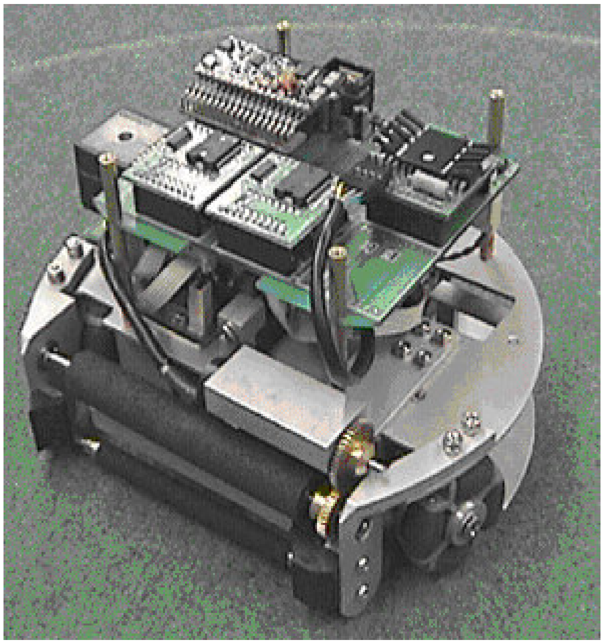

Figure 3: Robot

$$\begin{aligned} v_3 &= [-\sin(\alpha_3)\ \cos(\alpha_3)]^T \\ v_4 &= [0\ 1]^T \\ v_r &= [v_{rx}\ v_{ry}]^T \\ v_{rx} &= v_x \cos\theta + v_y \sin\theta \\ v_{ry} &= -v_x \sin\theta + v_y \cos\theta \end{aligned}$$

From equations (1) to (3), the angular velocities of the wheels are derived as the function of the robot linear and angular speeds,

$$\begin{aligned} \omega_1 &= r^{-1}(b\omega + v_{ry}\cos(\alpha_1) - v_{rx}\sin(\alpha_1)) && (5) \\ \omega_2 &= r^{-1}(b\omega + v_{ry}\cos(\alpha_2) - v_{rx}\sin(\alpha_2)) && (6) \\ \omega_3 &= r^{-1}(b\omega + v_{ry}\cos(\alpha_3) - v_{rx}\sin(\alpha_3)) && (7) \\ \omega_4 &= r^{-1}(b\omega + v_{ry}) && (8) \end{aligned}$$

Given the desired linear velocity $v = [v_x\ v_y]^T$ and the angular velocity ω, the required wheel speeds ω_i $(i = 1,2,3,4)$ can be determined through the above equations.

Remark 1: Only the kinematic modeling of the robot is presented in the above. The dynamic model of an omni-wheel robot is quite complicated [4]. A dynamic model based controller is normally very computationally intensive and is not suitable for a real time application. In this paper, the model free PID controller is used to design the speed controller for the robot.

3 Speed Control and Implementation

The aim of the motion controller is to make v and ω to reach their desired values v_d and ω_d respectively. Alternatively, the aim can be to make ω_i to achieve its desired value ω_{id} which is derived from v_d and ω_d $(i = 1,2,3,4)$.

The controlled variables can be those referred to the world coordinate of the robot (v and ω) or those refereed to each driving wheel ω_i $(i = 1,2,3,4)$. Taking ω_i as the controlled variable, the speed controller takes the form form of the commonly used PID controller

$$u_i = k_p(e_i + \frac{1}{T_i}\int_0^t e_i dt + T_d\dot{e}_i, \quad i = 1,\ 2,\ 3. \qquad (9)$$

where u_i is the control input (PWM values for the motor drive) for the ith wheel, $e_i = \omega_{di} - \omega_i$ and k_p, T_i and T_d are control parameters.

The speed controller is implemented on an Intel 80296 microcontroller. Each wheel of the robot is driven by a mini DC motor with which an encoder of 512 lines is attached for position feedback. The desired speed for each wheel is sent to the robot through a 900MHZ radio frequency wireless link from the host computer. The block diagram of the robot's embedded controller is schematically shown in Figure 4. The sampling time is set to 1 ms. To speed up the calculation in real time, most codes are written directly in assembly language. As the assembly language can only handle the whole numbers directly, a new representation of a number is built up so that a floating point number can also be processed with a reasonable accuracy. With only simple addition, subtraction, multiplication and bits-shift functions on integers provided by the assembly of 80296, the control input can be obtained with less than 0.5 ms time.

Though the dynamic model of the robot is not used in the controller design, some dynamic parameters such as the time constant of the system are needed for selecting the control gains through experiments. To get the proper values of k_p, T_i and T_d, some experiments are done to get the step response of v_x. Figure 5 shows the speed response when the desired speed v_x is 0.85 m/s. The position of the robot is captured by a vision system and it is processed further to obtain the velocity of the robot. From the response of v_x, we can estimate the time constant of the robot system and then adjust the control gains accordingly. It is obvious that the speed response of the robot is similar to that of 1st order system or critically damped second order system. The time constant of the overall system (the robot, the vision system and the wireless communications) is about 90 ms and it is important to tune the controller parameters and the trajectory planning of the robot.

The four-wheel omni directional mobile robot is tested experimentally. It is controlled to move along a straight line between two points $(149, 333)$ and $(468, 136)$ and a circle of a diameter of $100mm$ on a standard filed for RoboCup F180 category. The posture of the robot is captured by processing the robot images captured through a camera hanging above the field. The line and the circular trajectories traced by the robot are plotted in Figures 6 and 7 respectively. It can be seen that the movements of the robot are quite satisfactory.

Remark 2: The robot described in this paper is developed from the three-wheel omni-wheel robots used in our robotic soccer team - "TH-SP Field Rangers", a quarter finalist in the RoboCup 2004 [5]. The performance of that robot in terms of its motion was very good in the real game.

4 Conclusion

This paper presents the design, analysis and real time control of a four-wheel omni directional mobile robot. The kinematic model, speed control and implementation are described.

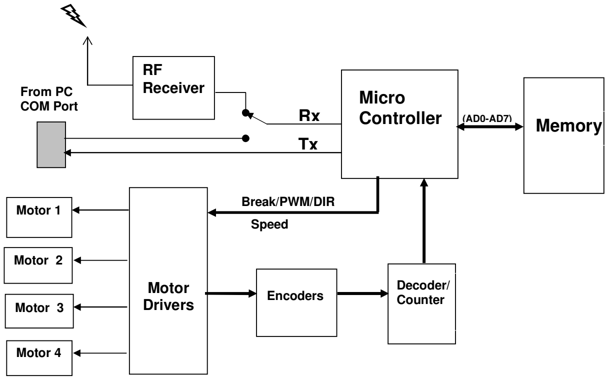

Figure 4: Block Diagram of the Controller

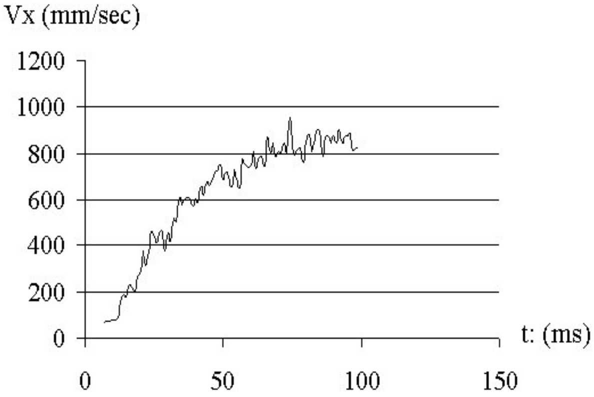

Figure 5: Response of Velocity v_x

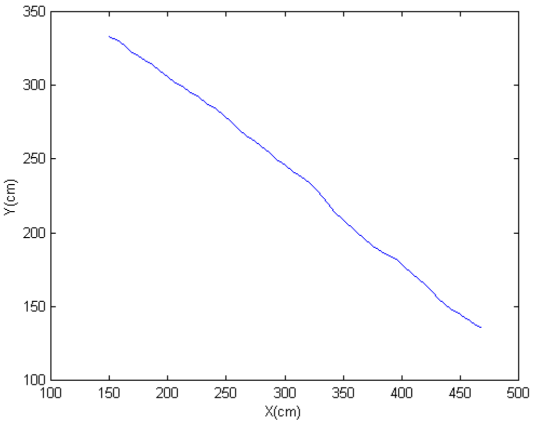

Figure 6: A Line Traced by the Robot

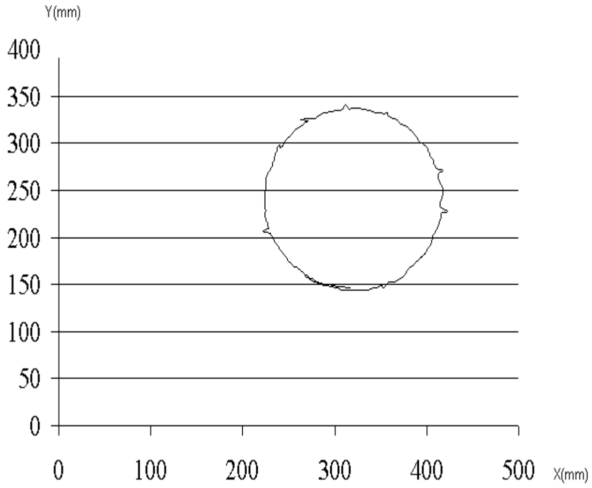

Figure 7: A Circle Traced by the Robot

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