2nd International Conference on Autonomous Robots and Agents
December 13-15, 2004 Palmerston North, New Zealand

Control and Stability Analysis on Multiple Robots Formation

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Abstract

It is difficult to completely and accurately derive modeling parameters of individual robots in performing a task. To achieve a formation task, the relative positions among robots are key factors. Through modifying individual dynamic models, we propose an adaptive NN control law with robust terms to control individual motions. By theoretical analysis we prove that all robots will form a unique formation with a few of communications among robots. Simulations are made to execute a group of robots construct a formation.

Keywords: formation, nonholonomic, adaptive NN, leaders-follower cell.

1 Introduction

In recent years, formation has been a very hot topic in robotics field. Briefly, formation task refers to two issues: 1) how to make a group of non-related robots form a formation; and 2) given a predetermined relationship among robots, how to keep a certain order in parade. The second topic will be addressed in this paper. For practical purpose, most robotic systems found in literatures on formation are mobile robots. some control laws are designed to make a group of nonholonomic robots keep a certain formation on an assumption of existing a predetermined leader for coordination[1] [2], and the stability of the system is proved based on individual control method. A behavior-based approach to formation maneuvers for groups of mobile robots is presented [3]. Complex formation maneuvers are decomposed into a sequence of maneuvers between formation patterns. The definition of LFS(leader-to-formation stability) is proposed to describe the performance of formation [4], then it is easy to use a quantitative value to measure the stability of formation for a multi-layer system. Of course, for an individual robot in a system, keeping a certain order can also be comprehended as a kind of tracking trajectory, i.e. the robot should track a moving reference point determined by other robots. In most literatures, individual control laws refer to kinematic models of mobile robots. In our paper, we will discuss a more practical situation in case of unknown individual model parameters and existing perturbations, how to apply adaptive NN control strategy based on dynamic model to control robots keeping in regular formation [5]. Based on our previous results and the individual control law [6] [7], we also discuss the integrated performance of the system by proving that all robots will form a unique formation which is determined by the formation leader and predetermined formation pattern.

2 Description of Formation

In general, formation task requires a group of robots to move together and keep a regular formation as well. In macro scope, the system is a decentralized control system since all robots decide their motions independently. We assume that all robots have identical sensor ability, and there is only one communication, i.e. the leader broadcasts its angle to other members to denote reference mark of the formation pattern.

The main problem for the task is how to describe the ideal formation and how to let robots know the relationship among them. There are many methods to describe this relationship, such as graph theory and leader-follower pairs. Here a kind of extended leader-follower pair is exploited to describe the formation structure.

To be convenient for analysis, some definitions are given below:

Definition 1(Leaders-follower Cell): Given a robot i, a set Ω_i is defined as leaders set of robot i, if robot i follows a reference point which is resulted from positions of robots in a set of Ω_i. Hence the combinations of robot i and all leaders in Ω_i are called leaders-follower cells. And such reference point can be expressed as

$$p_i^d = \sum_{j \in \Omega_i} S_{ij}(p_j + D_{ij}^d), \qquad (1)$$

Where $p = [x \quad y]^T$, $S = \{S_{ij}\}$ is a projection vector that describes how leaders affect the value of reference point. It's hold that $\sum_{j\in\Omega_i} S_{ij} = 1$. D_{ij}^d represents the ideal reference distances that robot should keep from each leaders. Moreover, we define the size of a cell equals the number of leaders in leader set Ω_i. Obviously, for a N robots formation, there are $N-1$ leaders-follower cells. In fact, such leaders-follower cell describe the local relationship among robots. Hence no robot knows all relationship among all members.

Definition 2(Formation): Given a group of robots, a formation can be defined as a combination of a group of leaders-follower cells. Because leaders of one cell can be followers of other cells, the shape of the ideal formation can be uniquely decided by cells. There must exist at least one robot that plays the role of leader, which can be defined as the leader of the formation. Here, we suppose that there is only one leader in the formation.

Definition 3(Leader-to-follower-link): In a Leaders-follower Cell, the relationship between a leader and its follower is defined as a leader-to-follower-link. Its direction is from leader to follower.

Definition 4(Link between two robots): Between two arbitrary robots, if there exists a set of leader-to-follower-links that form one and only one directional link connecting these two robots, we call this link as a link between two robots. The number of leader-to-follower-links contained in the link is defined as the size of the link.

To avoid the conflict of reference point, we define that given robot i and j no two links whose directions are $i \to j$ and $j \to i$ simultaneously.

Definition 5(Layer of formation): Between the leader of the formation and another robot, there must exist one or several links. We define the maximum of the sizes of these links as the layer where robot is. And we prescribe the leader of the formation is in layer zero.

An example of a formation pattern that is formed by four robots is shown in figure 1. There are three leaders-follower cells. Obviously, not all cells have the same size. For robot 3, there are two leaders, robot 1 and 2, and two leader-to-follower-links, III and IV, to form a leaders-follower cell. Further, robot 3 is in layer 3, and robot 1 and robot 2 are in layer1 and layer 2 respectively.

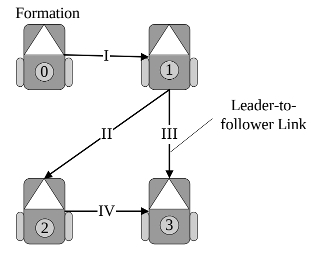


Figure 1: Sample of formation pattern.

3 Individual Control Strategy

In this paper, a kind of car-like robots are used to accomplish the task. The following assumptions are made: 1) All robots are two-wheel driven mini cars as shown in figure 2; 2) There are no communications among robots. 3) There are no sensor constraints for each robot. That means robot can perceive its leaders completely.

3.1 Dynamic Modeling

The general coordinates of a robot moving on a plane can be defined as $q = [x \quad y \quad \theta]^T$. In formation task, what we care about is how to make all robots keep predefined relative positions. That means the most concerning part is position $[x \quad y]^T$ in general coordinates.

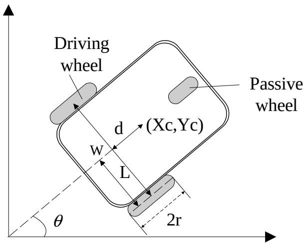


Figure 2: The two-wheel-driven mobile robot.

For a car-like robot shown in figure 2, we take the mass center as the robot's position, then the dynamic equation of robot can be expressed as

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + \tau_d = J^T(q)\lambda + B(q)\tau, \tag{2}$$

where $M(q) \in R^{3\times3}$ is a symmetric, positive definite inertial matrix, $V(q,\dot{q}) \in R^{3\times3}$ is the centripetal and coriolis matrix, $J(q) \in R^{3\times1}$ is the matrix associated with the nonholonomic constraints. $B(q) \in R^{3\times2}$ is the input transformation matrix. All these matrices are given by

$$M(q) = \begin{bmatrix} m & 0 & md\sin\theta \\ 0 & m & -md\cos\theta\,, \\ md\sin\theta & -md\cos\theta & I_0 + md^2 \end{bmatrix},$$

$$\tau = [\tau_l \quad \tau_r]^T, V(q,\dot{q}) = \begin{bmatrix} 0 & 0 & md\dot{\theta}\cos\theta \\ 0 & 0 & md\dot{\theta}\sin\theta \\ 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ L & -L \end{bmatrix}, J(q) = [\sin\theta \quad -\cos\theta \quad d].$$

In equation (2), τ_d is a torque which represents bounded disturbance and unmodeled dynamics.

Normally, the nonholonomic constraints can be expressed as $J(q)\dot{q} = 0$, or,

$$\dot{x}\sin\theta - \dot{y}\cos\theta + d\theta = 0. \tag{3}$$

From constraint equations, we know that the mass center is regarded as position coordination. Therefore, from equation (3), we can express the second order derivative of θ as:

$$\ddot{\theta} = \frac{1}{d}(\ddot{y}\cos\theta - \ddot{x}\sin\theta) + \frac{1}{2d^2}(\dot{x}^2 - \dot{y}^2)\sin 2\theta - \frac{1}{d^2}\dot{x}\dot{y}(\cos^2\theta - \sin^2\theta). \quad (4)$$

We can easily find a full rank matrix $S(q)$ which is formed by the vectors spanning the null space of constraint matrix $J(q)$ below.

$$S^T(q)J^T(q) = 0, \quad (5)$$

where $S(q) = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix}$.

Multiplying both sides of equation (2) with S^T to eliminate nonholonomic constraint forces, λ, we can rewrite the dynamic equation as

$$S^T M(q)\ddot{q} + S^T V(q,\dot{q})\dot{q} + \bar{\tau}_d = S^T B(q)\tau, \quad (6)$$

where $\bar{\tau}_d = S^T\tau_d$.

On the left side, the first two terms are rewritten as

$$S^T M\ddot{q} + S^T V\dot{q} = \begin{bmatrix} m\ddot{x}\cos\theta + m\ddot{y}\sin\theta + md\dot{\theta}^2 \\ I\ddot{\theta} \end{bmatrix}$$
$$= \begin{bmatrix} m\cos\theta & m\sin\theta \\ -\frac{I}{d}\sin\theta & \frac{I}{d}\cos\theta \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + (\dot{x}\sin\theta - \dot{y}\cos\theta) \begin{bmatrix} \frac{m}{d}\sin\theta & -\frac{m}{d}\cos\theta \\ \frac{I}{d^2}\cos\theta & \frac{I}{d^2}sin\theta \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix},$$

where $I = I_0 + md^2$.

Then we define a new coordinate $p = [x \quad y]^T$ which only denotes the position of robot. And further define $T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, $M_0 = \begin{bmatrix} m & 0 \\ 0 & \frac{I}{d} \end{bmatrix}$. Therefore, equation (6) can be rewritten as

$$M_0 T\ddot{p} + M_0\dot{T}\dot{p} = S^T B\tau - \bar{\tau}_d. \quad (7)$$

In equation (7), the description of robot's dynamics seems to be ignoring the heading angle. In fact, all information of robot is included in equation (7). While equation (4) describes the internal dynamics of robot.

3.2 Adaptive Neural Network Controller

For robot i, the ideal reference point is in the form of equation (1). Therefore, we define the relative position error as $e_i = p_i - p_i^d = p_i - \sum_{j\in\Omega_i} S_{ij}(p_j + D_{ij}^d)$. In this section, we will propose a controller to guarantee that $\lim_{t\to\infty} e_i = 0$. That means, for an individual robot, it can keep a unique relative distance from its leaders.

A filtered error is

$$z_i = \dot{e}_i + \Lambda e_i. \quad (8)$$

If define a temporal variable, $\dot{p}_i^r = \dot{p}_i^d - \Lambda e_i$, then, $z_i = \dot{p}_i - \dot{p}_i^r$. Substitute it into equation (7), we have

$$M_0 T_i\dot{z}_i + M_0\dot{T}_i z_i = S^T B\tau_i - \bar{M}_i\ddot{p}_i^r - \bar{V}_i\dot{p}_i^r - \bar{\tau}_{id}. \quad (9)$$

Let a new error be $\tilde{z}_i = T_i z_i$. Hence, equation (9) is transformed to

$$M_0\dot{\tilde{z}}_i = S^T B\tau_i - \bar{M}_i\ddot{p}_i^r - \bar{V}_i\dot{p}_i^r - \bar{\tau}_{id}. \quad (10)$$

Normally, we can use input-output feedback method to design a control law as

$$\tau_i = \left(S^T B\right)\left(-K\tilde{z}_i + \bar{M}_i\ddot{p}_i^r + \bar{V}_i\dot{p}_i^r + Jsign(\tilde{z}_i)\right) \quad (11)$$

Where $K = diag\{k_1, k_2\}$ in which k_1 and k_2 are positive values. $Jsign(\tilde{z}_i)$ is a sliding mode term to restrain the disturbance τ_d. But in practice, it's hardly to determine $\bar{M}_i$ and $\bar{V}_i$, because it is difficult to measure distance between the mass center and the central point of two wheels d. Hence a neural network is used to model $\bar{M}_i\ddot{p}_i^r + \bar{V}_i\dot{p}_i^r + \bar{\tau}_{id}$ on-line.

We define a nonlinear function $f(X_i) = \bar{M}_i\ddot{p}_i^r + \bar{V}_i\dot{p}_i^r$ in which $X_i = [\dot{p}_i \quad \theta_i \quad \ddot{p}_i^r \quad \dot{p}_i^r]^T$. To simplify the expression, we omit the subscript i. We suppose that there exists a two-layer feedforward NN which can approximate $f(X)$, i.e.

$$f(X) = W^T\sigma(V^T X) + \varepsilon, \quad (12)$$

where $V \in R^{N_I\times N_H}$ represents the input-to-hidden-layer interconnection weights, $W \in R^{N_H\times N_O}$ represents the hidden-layer-to-output interconnection weights, where N_H, N_I and N_O are the numbers of neurons in the hidden layer, the input layer, and the output layer respectively. Here, the activation function $\sigma(\cdot)$ is in the form of sigmoid function, i.e. $\sigma(x) = \frac{1}{1+e^{-x}}$. ε is the NN functional approximation error.

The control purpose is to construct an NN function $\hat{f}(X)$ to estimate $f(X)$. Such estimation can be written as

$$\hat{f}(X) = \hat{W}^T\sigma(\hat{V}^T X), \quad (13)$$

where $\hat{W}$ and $\hat{V}$ are estimates of NN weights.

Let the estimated errors be $\tilde{f} = f - \hat{f}$, $\tilde{W} = W - \hat{W}$, $\tilde{V} = V - \hat{V}$. Define the hidden-layer output error as $\tilde{\sigma} = \sigma - \hat{\sigma} = \sigma(V^T X) - \sigma(\hat{V}^T X)$. Appling Taylor series expansion, we get

$$\sigma(V^T X) = \sigma(\hat{V}^T X) + \sigma'(\hat{V}^T X)\tilde{V}^T X + O(\tilde{V}^T X), \quad (14)$$

where $\sigma'(\hat{y}) = \frac{\partial\sigma(y)}{\partial y}|_{y=\hat{y}}$. Therefore,

$$\tilde{\sigma} = \sigma'(\hat{V}^T X)\tilde{V}^T X + O(\tilde{V}^T X). \quad (15)$$

Substituting approximate $f(X)$ into equation (10), we have

$$M_0\dot{\tilde{z}} = S^T B\tau - f(X) - \overline{\tau}_d$$
$$= S^T B\tau - W^T\sigma(V^T X) - \overline{\tau}_d + \varepsilon. \quad (16)$$

The input-output feedback linearization control technology is exploited to stabilize individual robot system. The control law can be expressed as

$$\tau = (S^T B)^{-1}(\hat{W}^T\sigma(\hat{V}^T X) - K\tilde{z} + \gamma), \quad (17)$$

where $K = diag\{k_1, k_2\}$ in which k_1 and k_2 are positive, γ is a robust control term to suppress the disturbance in equation (16). It should be pointed out that, different from sliding mode term in equation (11), the robust term here suppresses not only disturbance of unmodeled structure of dynamics, but also functional approximation error of NN.
Substitute control law into equation (16), and let σ and $\hat{\sigma}$ denote $\sigma(V^T X)$ and $\sigma(\hat{V}^T X)$ respectively. We obtain,

$$M_0\dot{\tilde{z}} = -K\tilde{z} - W^T\sigma + \hat{W}^T\hat{\sigma} + \gamma - \overline{\tau}_d + \varepsilon. \quad (18)$$

Adding and subtracting $W^T\hat{\sigma}$ and $\hat{W}^T\tilde{\sigma}$ respectively,

$$M_0\dot{\tilde{z}} = -K\tilde{z} - \tilde{W}^T\tilde{\sigma} - \tilde{W}^T\hat{\sigma} - \hat{W}^T\tilde{\sigma} + (\gamma - \overline{\tau}_d + \varepsilon). (19)$$

Considering the error system shown in equation (14) and equation (15), we have

$$M_0\dot{\tilde{z}} = -K\tilde{z} - \tilde{W}^T(\hat{\sigma} - \hat{\sigma}'\hat{V}^T X) - \hat{W}^T\hat{\sigma}'\tilde{V}^T X + s + \gamma, \quad (20)$$

where $s(t)$ is a disturbance term.

$$s(t) = -\tilde{W}^T\hat{\sigma}' V^T X - W^T O(\tilde{V}^T X) - \overline{\tau}_d + \varepsilon. \quad (21)$$

The adaptive backpropagation learning algorithm is expressed as

$$\begin{array}{l} \dot{\hat{W}} = F\hat{\sigma}'\hat{V}^T X\tilde{z}^T - F\hat{\sigma}\tilde{z}^T - \kappa F\|\tilde{z}\|\hat{W} \\ \dot{\hat{V}} = -GX(\hat{\sigma}'^T\hat{W}\tilde{z})^T - \kappa G\|\tilde{z}\|\hat{V} \end{array}, \quad (22)$$

where F and G are positive definite design parameter matrices governing the speed of learning.
Combining control law equation (17) and on-line tuning strategy equation (22), we can get a complete individual control strategy.

3.3 Stability of Individual Controller

In this section, we prove that under the individual control, the feedback system is stable. And the robust term is designed in order to prove the stability.To simplify denotation, we ignore the subscript i in this section. The Lyapunov function is defined as

$$L = \frac{1}{2}[\tilde{z}^T M_0\tilde{z} + tr\{\tilde{W}^T F^{-1}\tilde{W}\} + tr\{\tilde{V}^T G^{-1}\tilde{V}\}]. \quad (23)$$

Differentiating equation (23), and considering equation (20), we have

$$\dot{L} = -\tilde{z}^T K\tilde{z} + tr\{\tilde{W}^T(F^{-1}\dot{\tilde{W}} - \hat{\sigma}\tilde{z}^T + \hat{\sigma}'\hat{V}^T X\tilde{z}^T)\}$$
$$+ tr\{\tilde{V}^T(G^{-1}\dot{\tilde{V}} - X\tilde{z}^T\hat{W}^T\hat{\sigma}')\} + \tilde{z}^T(s+\gamma). \quad (24)$$

Substitute equation (22) into equation (24), and consider that $\dot{\tilde{W}} = -\dot{\hat{W}}$ and $\dot{\tilde{V}} = -\dot{\hat{V}}$,

$$\dot{L} = -\tilde{z}^T K\tilde{z} + \kappa\|\tilde{z}\| tr\{\tilde{W}^T(W - \tilde{W})\}$$
$$-\kappa\|\tilde{z}\| tr\{\tilde{V}^T(V - \tilde{V})\} + \tilde{z}^T(s+\gamma). \quad (25)$$

We define that $Y = diag\{W, V\}$. Accordingly, we also define that $\hat{Y} = diag\{\hat{W}, \hat{V}\}$, and $\tilde{Y} = Y - \hat{Y}$. Then, equation (25) can be expressed as

$$\dot{L} = -\tilde{z}^T K\tilde{z} + \kappa\|\tilde{z}\| tr\{\tilde{Y}^T(Y - \tilde{Y})\} + \tilde{z}^T(s+\gamma). \quad (26)$$

It is hold that $tr\{\tilde{Y}(Y - \tilde{Y})\} = \langle\tilde{Y}, Y\rangle_F - \|\tilde{Y}\|_F^2 \le \|\tilde{Y}\|_F\|Y\|_F - \|\tilde{Y}\|_F^2$.
And we define the robust term as

$$r = \begin{cases} -K_r(\|\hat{Y}\|_F + Y_M)\tilde{z} - J\frac{\tilde{z}}{\|\tilde{z}\|}, & \|\tilde{z}\| \neq 0 \\ -K_r(\|\hat{Y}\|_F + Y_M)\tilde{z}, & \|\tilde{z}\| = 0 \end{cases}, \quad (27)$$

where J and K_Y are positive. In following proof, we will provide the bound of these two values.
Substitute these two equations into equation (26). If $\|\tilde{z}\| \neq 0$, we get

$$\dot{L} \le -K_{min}\|\tilde{z}\|^2 + \kappa\|\tilde{z}\|(\|\tilde{Y}\|_F\|Y\|_F - \|\tilde{Y}\|_F^2)$$
$$-K_Y(\|\hat{Y}\|_F + Y_M)\|\tilde{z}\|^2 - J\|\tilde{z}\| + \|\tilde{z}\|\|s\|, \quad (28)$$

where K_{min} is the minimum singular value of K.
There is a fact that $\|s\| \le c_0 + c_1\|\tilde{Y}\|_F + c_2\|\tilde{Y}\|_F\|\tilde{z}\|$ [8], substitute it into equation (28),

$$\dot{L} \le -K_{min}\|\tilde{z}\|^2 + \kappa\|\tilde{z}\|(\|\tilde{Y}\|_F\|Y\|_F - \|\tilde{Y}\|_F^2)$$
$$-K_Y(\|\hat{Y}\|_F + Y_M)\|\tilde{z}\|^2 + \|\tilde{z}\|(c_0 + c_1\|\tilde{Y}\|_F$$
$$+c_2\|\tilde{Y}\|_F\|\tilde{z}\|) - J\|\tilde{z}\|, \quad (29)$$

Where Y_M is the bound of ideal weights.
It is hold that $\|\hat{Y}\|_F + Y_M \ge \|\hat{Y}\|_F + \|Y\|_F \ge \|Y - \hat{Y}\|_F = \|\tilde{Y}\|_F$. If we take $K_Y > c_2$, we can obtain

$$\dot{L} \le -K_{min}\|\tilde{z}\|^2 + \kappa\|\tilde{z}\|(\|\tilde{Y}\|_F\|Y\|_F - \|\tilde{Y}\|_F^2)$$
$$+\|\tilde{z}\|(c_0 + c_1\|\tilde{Y}\|_F) - J\|\tilde{z}\|$$
$$\le -\|\tilde{z}\|[K_{min}\|\tilde{z}\| - \kappa\|\tilde{Y}\|_F(Y_M - \|\tilde{Y}\|_F) - c_0$$
$$-c_1\|\tilde{Y}\|_F + J]$$
$$= -\|\tilde{z}\|[K_{min}\|\tilde{z}\| + \kappa\|\tilde{Y}\|_F^2 - \|\tilde{Y}\|_F(\kappa Y_M + c_1) - c_0 + J]$$
$$= -\|\tilde{z}\|[K_{min}\|\tilde{z}\| + \kappa(|\tilde{Y}\|_F - \frac{C_3}{2})^2 - \frac{\kappa C_3^2}{4} - c_0 + J], \quad (30)$$

Where $C_3 = Y_M + \frac{c_1}{\kappa}$.
Obviously, if we take $J \ge \frac{\kappa C_3^2}{4} + c_0$, we can obtain

$$\dot{L} \le -\|\tilde{z}\|\left[K_{min}\|\tilde{z}\| + \kappa(\tilde{Y}\|_F - \frac{C_3}{2})^2\right] \le 0. \quad (31)$$

According to LaSalle's principle, the system must stabilize to the invariant set $\{\tilde{z} \mid \dot{V} = 0\}$, where $\tilde{z} = 0$. We can easily prove that $z^T M_0 z = \tilde{z}^T M_0 \tilde{z}$. So the invariant set can also be expressed as $\{\tilde{z} \mid \dot{V} = 0\}$, where $z = 0$. Furthermore, e and $\dot{e}$ converge to zero too.
Based on the proof of stability, we can conclude that the adaptive NN control law with robust term can make robot follow the trajectory determined by its leaders.

4 Performance of Formation

The individual control guarantees that robots can follow their leaders. But during the process, even leaders in a leaders-follower cell do not converge to the stable states. In this section, we will prove that every robot will converge to a unique reference point that is determined by the formation's leader and combinations of desired distance.
Given a leaders-follower cell, assume that robot i is in the layer N. In its leader set of Ω_i, all its leaders must be in layers less than N. Let robot i's leaders set be $\Omega_i = \{\Omega_i^0, \Omega_i^1, \cdots, \Omega_i^{N-1}\}$, where Ω_i^k represents a subset of Ω_i, whose element denotes the robots in layer k.
The reference point of robot i can be written as

$$p_i^d = \sum_{n=0}^{N-1} \sum_{j\in\Omega_i^n} S_{ij} p_j + \sum_{n=0}^{N-1} \sum_{j\in\Omega_i^n} S_{ij} D_{ij}^d \tag{32}$$

For every leader j not in layer 0, we can express it as

$$p_j = e_j + p_j^d = e_j + \sum_{n=0}^{M-1} \sum_{k\in\Omega_j^n} S_{jk} p_k + \sum_{n=0}^{M-1} \sum_{k\in\Omega_j^n} S_{jk} D_{jk}^d. \tag{33}$$

where M represents the layer where robot j is.
Repeatedly applying equation (33) for any p_j on right side of equation (32) and considering for arbitrary robot l, $\sum_{m\in\Omega_l} S_{lm} = 1$, we get the final expression of robot i's reference point,

$$p_i^d = p_0 + \sum_{k\in\Xi_i} b_k D_k^d + \sum_{j\in\Psi_i} a_j e_j, \tag{34}$$

where Ξ_i represents the set whose elements are leader-to-follower-links which form links between the formation's leader and robot i, Ψ_i represents the set whose elements are robots on the links between the leader of formation and robot i, p_0 is the position of the formation's leader. a_j, b_k are constant coefficient matrices.
The position error of robot i is $e_i = p_i - p_i^d$. We define a new error

$$\tilde{p}_i = p_i - \left(p_0 + \sum_{k\in\Xi_i} b_k D_k^d\right). \tag{35}$$

Accordingly, the filtered error is $r = \dot{\tilde{p}}_i + \Lambda\tilde{p}_i$. Then, we have

$$z_i = r_i - \sum_{j\in\Psi_i} a_j \dot{e}_j - \Lambda \sum_{j\in\Psi_i} a_j e_j. \tag{36}$$

Substituting into equation (20), we have

$$M_i^0 \dot{\tilde{r}}_i = -K_i \tilde{r}_i - \tilde{W}_i^T(\hat{\sigma} - \hat{\sigma}'\hat{V}_i^T X_i) - \hat{W}_i^T \hat{\sigma}'\tilde{V}_i^T X_i + s_i + \gamma_i + u_i, \tag{37}$$

where $\tilde{r}_i = T_i r_i$,

$$u_i = T_i \sum_{j\in\Psi_i} a_j \ddot{e}_j + (T_i\Lambda_i + \dot{T}_i + K_i T_i) \sum_{j\in\Psi_i} a_j \dot{e}_j + (\dot{T}_i + K_i T_i)\cdot\Lambda_i \sum_{j\in\Psi_i} a_j e_j. \tag{38}$$

In equation (37), u_i is an input. In section 3.3, it has been proved that, for arbitrary individual robot i, the feedback dynamics is asymptotic stable. Therefore, there must exist a class $\mathscr{KL}$ function $\phi(\cdot)$, so that for any error $e_j, j \in \Psi_i$, $\|e_j(t)\| \le \phi(\|e_j(0)\|, t)$. If it is hold that, $\|T_i\|$, $\|T_i\Lambda_i + \dot{T}_i + K_i T_i\|$ and $\|(\dot{T}_i + K_i T_I)\cdot\Lambda_i\|$ are bounded, there must exist a positive value u_i^M so that $\|u_i(t)\| \le u_i^M$. At the same time, because $\lim_{t\to\infty} e_j = 0, j \in \Psi_i$, we have $\lim_{t\to\infty}\|u_i(t)\| = 0$.
Assume that there are N robots, define $P = [p_0 \ p_1 \ \cdots \ p_{N-1}]^T$, $P^d = [p_0^d \ p_1^d \ \cdots \ p_{N-1}^d]^T$, $R = (\dot{P} - \dot{P}^d) + \Lambda(P - P^d)$, where $\Lambda = diag\{\Lambda_i\}, i = 0, \cdots, N-1$. Define a positive differentiable function as

$$L = \tfrac{1}{2}\left[(\bar{T}R)^T \bar{M}(\bar{T}R) + tr\{\bar{\tilde{W}}^T \bar{F}^{-1}\bar{\tilde{W}}\} + tr\{\bar{\tilde{V}}^T \bar{G}^{-1}\bar{\tilde{V}}\}\right], \tag{39}$$

where $\bar{T} = diag\{T_i\}$, $\bar{M} = diag\{M_i^0\}$, $\bar{\tilde{W}} = diag\{\tilde{W}_i\}$, $\bar{\tilde{V}} = diag\{\tilde{V}_i\}$, $\bar{F} = diag\{F_i\}$, $\bar{G} = diag\{G_i\}$, $i = 0, \cdots, N-1$. In fact, $\bar{M}$, $\bar{F}^{-1}$, $\bar{G}^{-1}$are diagonal positive defined matrices. Then, it must hold

$$\alpha_1(R, \bar{\tilde{W}}, \bar{\tilde{V}}) \le L \le \alpha_2(R, \bar{\tilde{W}}, \bar{\tilde{V}}), \tag{40}$$

where $\alpha_1(\cdot)$, $\alpha_2(\cdot)$ are class $\mathscr{K}$ functions.
If we adopt the individual control law shown in section 3, and apply the results shown in equation (31), we will obtain

$$\begin{aligned}\dot{L} &\le -\tilde{R}^T \bar{K}_{min}\tilde{R} + \tilde{R}^T U - \sum_{i=0}^{N-1} \kappa\|\tilde{r}_i\|\left(\|\tilde{Y}_i\|_F - \tfrac{C_3}{2}\right)^2 \\ &\le -\|\tilde{R}\|\left(\bar{K}_{min}\|\tilde{R}\| - \|U\|\right) - \sum_{i=0}^{N-1} \kappa\|\tilde{r}_i\|\left(\|\tilde{Y}_i\|_F - \tfrac{C_3}{2}\right)^2,\end{aligned} \tag{41}$$

where $\tilde{R} = \bar{T}R$, $U = [u_0 \ u_1 \ \cdots \ u_N - 1]^T$. Obviously, $u_0 = 0$.
Of course we can not guarantee $\forall t > 0$, $\bar{K}_{min}\|\tilde{R}(t)\| \ge \|U(t)\|$. But since it is hold that $\lim_{t\to\infty}\|U\| =$

$\lim_{t\to\infty}\left(\sum_{i=1}^{N-1}\|u_i\|^2\right)^{0.5} = 0$, there must exist T_M, so that $\forall t \geq T_M$, $K_{min}\|\tilde{R}(t)\| \geq \|U(t)\|$. Therefore, we can find a class $\mathscr{K}$ function $\alpha_3(\cdot)$ such that $\dot{L} \leq -\alpha_3(R,\tilde{W},\tilde{V})$. According to the theorem of input-to-state stability [9], there exist a class $\mathscr{K}\mathscr{L}$ function $\beta(\cdot)$ and a class $\mathscr{K}$ function $\gamma(\cdot)$, which make solution $\tilde{R}(t)$ satisfy

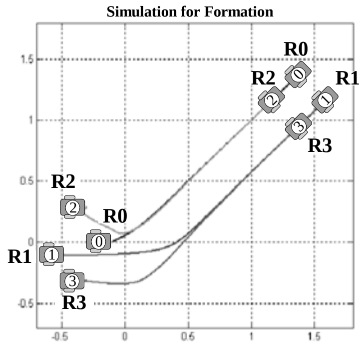
$$\|\tilde{R}(t)\| \leq \beta(\|\tilde{R}(T_M)\|, t-T_M) + \gamma\big(\sup_{T_M\leq\tau\leq t}\|U(t)\|\big). \quad (42)$$

Furthermore, since $\lim_{t\to\infty}\|U\| = 0$, when $t \to \infty$, equation (42) reduces to

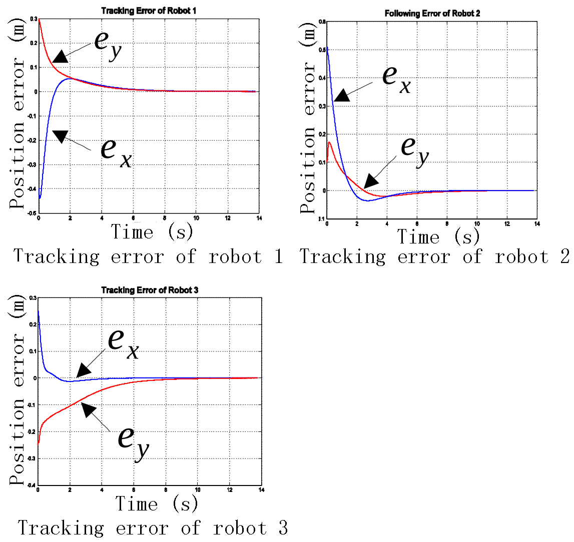
$$\|\tilde{R}(t)\| \leq \beta(\|\tilde{R}(T_M)\|, t-T_M). \quad (43)$$

It's easy to prove that $\|R(t)\| = \|\tilde{R}(t)\|$. So, we can conclude that, the system is globally uniformly asymptotically stable. From equation (35), we know that, given robot i, in the end, it follows the reference point $p_0 + \sum_{k\in\Xi_i} b_k D_k^d$ without error. That means, for every robot, the reference point is uniquely determined by the leader of the formation and combinations of desired relative distances.

The result of formation simulation is shown in Figure 3, in which 3(a) four robots are scattered within the arena. In the end, four robots form a regular order. Figure 3(b) shows the tracking errors of robot 1 through robot 3. Notation e_x and e_y denote the tracking errors along X axis and Y axis respectively. We observe that all robots can track their reference points at last.



(a)



(b)

Figure 3: Simulation result.

5 Conclusion

The leaders-follower cell is defined to describe relationship among robots in this paper. Such description is uniquely determined by the leader of the formation and desired relative positions. And the control strategy based on adaptive NN can guarantee that all robots will form the formation as desired.

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