2nd International Conference on Autonomous Robots and Agents
December 13-15, 2004 Palmerston North, New Zealand

When Physics Rules Robotics

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Abstract

We consider some fundamental issues that arise as we move in the directions of robots that are much larger or much smaller than everyday human scale. In some interesting cases both much larger and much smaller prevail simultaneously, e.g., large networks of small devices. Much of the discussion focuses on scaling issues that were first recognized in biological versus technological domains; these are corresponding discussed from the biological perspective, under the assumption that the reader will find it natural to transpose the underlying principles to technological domains in general, and to the robotics domain specifically. After a brief discussion of mechanical scaling issues – all in principle well known to robot designers – we examine energy and power related issues, focusing primarily on the inevitable range and running time limitations of micro- and nano-robots.

Keywords: robotics, scaling, energy, power, communication

1 Introduction

1.1 Background

Scaling was the first of the *Two New Sciences* revealed in Galileo's *Discourses and Mathematical Demonstrations* (1638); physics was the second[1][2]. The practical connection between them is materials: *big is weak, small is strong* is a consequence of the impossibility of altering the strength of matter in parallel with altering the size of the structures made of that matter. Galileo understood this: "... the mere fact that it is matter makes the larger machine, built of the same material and in the same proportion as the smaller, correspond with exactness to the smaller in every respect except that it will not be so strong ... who does not know that a horse falling from a height of three or four cubits will break his bones, while a dog falling from the same height ... will suffer no injury?" Similarly, the ultimate inalterability of achievable energy storage density – also a consequence of the fundamental strength of matter – links every mobile machine's range to its size, profoundly limiting the prospects for building arbitrarily small robots that will operate in arbitrarily low available energy environments. The large and small ends of robotics come full circle in the development of large networks of small robots, where geometrical scale issues again both enable and constrain the practicality of the internal communications essential to network functionality.

Two generalities, both at first counterintuitive but both straightforwardly physics-based, rule the design of both living and engineered structures and devices: (1) *big is weak, small is strong*, i.e., it is large structures that collapse under their own weight, large animals that break their legs when they stumble, etc., whereas small structures and animals are practically unaware of gravity, and (2) horses *eat like birds* and *birds eat like horses*, i.e., a large animal or machine stores relatively larger quantities of energy and dissipates relatively smaller quantities of energy than a small animal or machine. The critical consequence of (1) is that it is hard to build large structures and easy to build small structures that that easily support their own weight. The critical consequence of (2) is that it is hard to build small structures and easy to build large structures that easily operate long enough and travel far enough to do any sort of interesting job.

1.2 Strength

Strength related scaling is not yet much of a problem in robotics. Big-end robots – e.g., radio telescopes – are designed by mechanical engineers who know how to build structures that only rarely collapse under their own weight (see figure 1). And the mechanical over-design of the present generation of small-end robots – e.g., prototype fly-on-the-wall nano-robot spies – does not significantly decrease their already miniscule functionality. Over design of small machines helps relax some manufacturing challenges; it is apparent even in robots built to near-human scale, e.g., Honda's tour-de-force humanoid Asimo, whose body proportions are those of a three- or four-meter man. This unnatural scaling causes disturbing perceptual dissonance when Asimo's actual 1.2 meter height is revealed by pictures of him with humans (see figure 2). EPFL's Alice is an example of a state-of-the-art over-designed mini-robot; it employs a geometrical scale that seems appropriate to a much larger human scale vehicle, e.g., a wheelchair (see figure 3).

Figure 1: A large robot that later collapsed under its own weight. [Image courtesy of NRAO/AUI; see http://ftp.gb.nrao.edu/imagegallery.]

Figure 2: Perceptual dissonance due to over design of Honda's ASIMO Humanoid.
[From http://www.honda.co.jp/ASIMO/technology/tech_09.html]

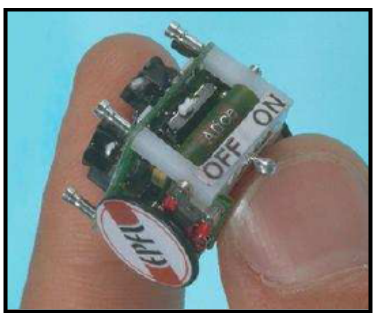

Figure 3: EPFL's Alice mini-robot, based on a watch motor. [Floreano et al, Evolutionary Bits 'n' Spikes, Artificial Life VIII, MIT 2002, pp. 335-344.]

1.3 Energy

The most serious scaling problem for present day robotics relates not to strength but to energy: universal enthusiasm for applications of tiny robots is untempered by the should-be-obvious fact that a bug cannot pack in enough calories to do much more than look for its next meal. The temporal endurance of any machine is its stored energy divided by its minimum power requirement. Stored energy obviously scales as the cube of a characteristic length. There are innumerable scenarios for minimum power, several of them analyzed in some detail in Section 3. As discussed in Section 3.8, the most useful model for a machine whose purpose is to move is probably that drag is proportional to the product of frontal area and velocity. Time-between-meals is thus proportional to length divided by velocity, and range is proportional to length. With step size proportional to characteristic length, all machines have the same range in steps and the same running time in step times. From the same sort of argument – even for machines with very different minimum power models – it invariably emerges that small robots on useful missions must either run on energy beamed in from the outside or must forage for it in their environments. It's a good thing too, otherwise the air we breathe would probably be as densely populated with micro-organisms as is the energy-rich liquid environment running through the sewers under our feet. Apropos of this observation, a sewage-powered robot was recently described [3].

1.4 Communication

The public is fascinated by visions of smart micro-robots; the roboticists are fascinated by visions of huge armies of not-so-smart nano-robots organizing themselves into super-brains and mega-bodies that adapt themselves to any task. Robots were classically defined as machines that sense, think, and act. When roboticists realized that what makes robots interesting is their mobility I added communicate to my personal version of this paradigm.

Societies of many robots will need to communicate with each other even more than they will need to communicate with us. High-density robot societies – those in which inter-robot distance is typically no more than a few tens of robot characteristic dimensions – will be able to use the same sorts of one-spatial-dimension communication channels we use in our bodies, our machines, and most of our telecommunications. But low density societies of highly mobile individuals – those like the contemplated global environmental monitoring network, 10^{10} nodes seeded 1 per km^3 to an altitude of 20 km over the surface of the earth – will need somehow to contend with $1/r^2$ communications in an uncertain direction at least for signal acquisition, and – unless unlikely sophisticated pointing technology emerges – nodes in a turbulent viscous medium will probably need always to broadcast into large solid angles. Intriguing solutions can be contemplated via device scales that are macroscopic in some dimensions and microscopic in others, e.g., decimeter-long filaments of deka-micron diameter, making them good antennas – and good sails – whose volume nevertheless fits into the 1-mm cube that is the practical upper size limit for manufacturing 10^{10} devices without making impossible demands on the world's annual production of silicon wafers.

1.5 Scope

When I use the term "fundamental issues", , e.g., in the abstract, I mean opportunities provided by and restrictions imposed by the most basic laws of physics as they relate to things like the strengths of structures, the internal and external motions of the structures, the energy requirements associated with their basal metabolisms, the mechanical work they do, the energy they dissipate to friction associated with their mobility and the work they do, as well as some communication issues relating to energy cost and signal range, and the relationship between the size of an antenna and the efficiency with which it couples to the environment at any particular communication frequency.

This being an exercise in reality, what I will not consider is hypothetical possibilities that are contingent on finding construction materials, energy storage principles, operating environments, etc., whose physical parameters, differ substantially from materials that actually exist or that we can realistically imagine developing. On the other hand, it is sensible for us to consider environmental parameters that are outside familiar ranges, e.g., extra-terrestrial, subterranean, deep-ocean, etc., environments in which temperatures, pressures, gravitational acceleration, etc., would be very different. Although I will not discuss the consequences of any of these in detail, it should be apparent that different environmental parameters lead to different expectations.

1.6 Organization

The subsequent sections provide additional discussion and analysis (Sections 2 and 3), conclusions (Section 4), acknowledgements (Section 5), and references (Section 6). Section 2, begins with subsections on size and strength (2.1), energy (2.2), and force (1.3)), the later introducing some dynamic issues that are familiar in animal efficiency studies but not yet particularly relevant to robotics because current generation robots are generally over designed in the strength domain and underperforming in the dynamics domain. Section 3 discusses a hypothetical family of robot vacuum cleaners that differ from each other in scale. It addresses performance – primarily range and operating time on stored energy – in several alternative maintenance power scenarios including one dominated by the power needed to move air (3.3), one dominated by the power needed to overcome brush friction (3.4), a "constant cleaning power" model (3.5), and a power loss to body drag model (3.6), argued in Sections 3.7 and 3.8 to be the most relevant for general robotic vehicle scenarios.

2 Size, Strength, Energy, and Force

Good examples of robots that are much larger than human scale are radio telescopes like the one shown in figure 1, extraterrestrial structures like the International Space Station, and modern buildings that incorporate large dynamic elements that actively compensate for wind and earthquake forces.

Good examples of robots that are much smaller than human scale are the mobile devices contemplated for applications like exploring and treating ailments of the human body from the inside out, dust-particle-sized active nano-sensors for global scale environmental monitoring, and the micro-scale active components of advanced airfoil surfaces.

2.1 Strength

Interesting issues with important practical consequences arise even for size – and consequent strength – decisions about structures that are only a little different from human-sized. For example, I would be inclined to make my entry in DARPA's Grand Challenge [4] a shoebox or smaller sized vehicle. It would give me extra leeway for staying on the road and passing obstacles, it would be rugged in a tip-over, it would be easy to right if it did tip, and it would be difficult for an adversary to detect and target. But my strategy has a fatal flaw: a jeep-sized vehicle can easily carry enough fuel to cover the 200 km course – and return home too – but even an extremely efficient shoe-box sized vehicle would be hard pressed to cover just a few kilometers.

The occasional collapse of radio telescopes, bridges, and medieval cathedrals notwithstanding, it is scales smaller than ordinary human experience that typically thwart robotics applications by constraining robot run time and range. We will subsequently demonstrate quantitatively in several maintenance-power scenarios that the unavoidably limited energy carrying capacity of small structures requires that, below a critical size it inevitably becomes necessary to extract energy more-or-less continuously from the environment vs. carrying energy for the duration of the mission.

2.2 Energy

We can acquire an intuitive feeling for the absolute scales at which energy carrying capacity becomes, at the small end, an insurmountable barrier, and, at the large end, an issue only at intercontinental distances, by looking at some examples from the animal world at the small end and some examples from the engineering world at the large end.

At the small end, fly-sized insects crawl and even fly substantial distances between feedings, but mites that get down to barely visible size are pretty much constrained to live parasitically on food-bearing surfaces. Bacteria-sized microorganisms usually perish rapidly – the germ-fear-exploiting advertising of household cleanser purveyors notwithstanding – when removed from the energy-rich three dimensional soup in which they are normally bathed.

At the large end we recognize that long distance transportation is most economically provided by a small number of very large vessels vs. a large number of very small vessels, fuel capacity inevitably winning over the many other considerations – some mentioned previously – that favor smaller vehicles. Absolute scales at this end are already quite intuitive to us:

small airplanes and large cars have ranges in the 500 km regime, medium sized airplanes can cross the Atlantic economically, but only the largest airplanes – often retrofitted with auxiliary fuel bladders – can cross the Pacific expanse. Like small organisms, small boats can travel large distances only by foraging for energy en route, e.g., by sailing; they can carry enough fuel only for maneuvering in port and for short excursions. A petroleum tanker, on the other hand, could probably run on its own fuel load until it wore out its engines.

2.3 Force

The foregoing considers the factors that determine how big big structures can be before there is no material strong enough to keep them from collapsing under their own weight – if you don't believe it ask yourself why planet-size objects are invariably near-spherical – and how small small structures can be before there is no energy storage medium dense enough to sustain a useful run time. Structural integrity and the energy to get from here to there are both crucial, but neither says more than a little about the ability to do useful mechanical work. Of course many useful robotic tasks can be accomplished without doing any mechanical work beyond what it takes to get from here to there; for example, they can simply carry sensors that convey enormously valuable data to remotely located people. Still, if for no reason but completeness, it is important to ask and understand what matters in this respect.

Static scaling issues have been discussed since the dawn of modern science, but dynamic scaling issues – relating mostly to how fast animals can run, how high they can jump, etc., – seem not to have been discussed until A. V. Hill's *The dimensions of animals and their muscular dynamics* [5] was published in 1950, though Hill had laid the groundwork in 1938 when he published *The heat of shortening and the dynamic constants of muscle* [6], excellently summarized by Pennycuick's *Newton Rules Biology: A physical approach to biological problems* [7]. The interesting constraint across the entire animal kingdom is that all muscle is essentially the same, and only a small variety of energy sources, range-of-motion transformers, and power integrators are available to animals.

In contrast the forces that electromechanical actuators can exert, their ranges-of-motion, and their speeds are very flexible: mechanical and electrical transformers can convert between whatever the prime mover delivers and whatever the application requires. Power issues per se are also relatively minor for robots, since energy from a low power source can usually can be integrated by springs or capacitors and delivered as rapidly as may be required, albeit for a limited time.

To efficiently drive a repetitive motion – a flapping wing or a running leg – it is necessary for the period of the muscle action, the length of the muscle divided by a limiting velocity characteristic of the muscle material, to match the period of the motion. Also recognizing that when these motions are efficient they are essentially pendulous – for terrestrial but not for aquatic animals – it can be shown that that the cruising speed of geometrically similar animals – e.g., members of the cat family – increases with their size, but their top speed running flat out is independent of size. This is confirmed by observational data. Similar considerations lead to the conclusion that all animals – actuated, as stated, by essentially identical muscle material – should be able to jump to the same height. The flea, often credited with being the world's champion jumper, actually does much worse than this analysis suggests, primarily because it is too small to push against the floor long enough to realize its potential. Robotic mechanisms have more leeway than animals because they are not constrained to use one kind of muscle for every job.

3 Robot Vacuum Cleaner Family

We can imagine a family of robotic vacuum cleaners, all of the same design, but implemented at various scales from the huge aircraft hangar model down to the standard residential model, then further down to the mouse-sized model for cleaning under furniture, and even further down to the ant-sized model for cleaning, say, the crevices between bathroom tiles. By focusing on this hypothetical family of robots related to each other only by scale we can pose a broad set of questions whose answers provide us with comprehensive quantitative insight.

3.1 Energy, Power, and Running Time

The quantitative relationship between size and energy carrying capacity is easy: for any given energy storage medium – batteries, liquid fuel, etc. – the stored energy increases as the volume, i.e., as the cube of the linear dimension h.

The running time between recharging, refueling, etc., is thus proportional to h^3/P, where P is the power demand, i.e. time = energy/power.

There are innumerable scenarios for how P might scale with h. A simple model that is adequate to introduce the topic is to say that it is simply proportional to the robot's surface area h^2, from which we conclude that the machine's potential running time is proportional to h. In this particular scenario a robotic vacuum cleaner design that runs for 30 minutes when its diameter is 30 cm could run for only 1 minute when its diameter is reduced to 1 cm. We will subsequently consider several alternative models for the dependence of P on h, how they play out, and what can be concluded from the outcomes.

3.2 Baseline Energy Demand

First, under what circumstances is the initial illustrative assumption that P is proportional to h^2 the correct model? If the vacuum cleaner is not a vacuum cleaner but, say, a mouse, then the body heat loss rate to the environment, i.e., the power required to maintain the body temperature, is proportional to h^2, and the time during which the stored energy can keep the mouse above any specified threshold temperature is proportional to h. The same is true if the on-board energy is expended to keep the mouse – or robot – cool, e.g., it absorbs energy from solar illumination or from a hot atmosphere that it needs to dissipate to prevent overheating of its delicate organs or electronics. An on-board heat pump must operate at a rate proportional to the surface area, h^2, hence the time during which stored energy can keep the heat pump running is proportional to h. Of course, exactly how to do this – perhaps by making a part of the surface a radiator that is actually hotter than the surroundings in order to cool the bulk of the volume, or by sucking in some of the hot atmosphere, making it even hotter, and expelling it – might be an engineering challenge, but it is certainly possible.

3.3 Energy Lost Moving Air

So what is the right model for a vacuum cleaner – a machine that needs to cover some ground vs. an animal that needs to keep itself warm or cool? It may depend on how efficient the vacuum cleaner is. If it really is a “vacuum cleaner”, an awfully inefficient machine that wastes most of its power on blowing air and making noise, and if the important issue is to maintain a constant air velocity at the intake irrespective of the machine’s scale, then the power required is again proportional to h^2, and the running time is still proportional to h. This assumes that we don’t make it so small that its dimensions become comparable to the mean-free-path of the air molecules, in which case it would probably not be possible to satisfy the goal of maintaining an arbitrary air intake velocity.

3.4 Energy Lost to Brush Friction

So what if it is a more efficient sort of “vacuum cleaner”, one that actually picks up dirt with a rotating brush rather than by sucking it in with a high-velocity air flow? Most of the power might then be expended in the friction of the brush on the floor. To determine the running time we must ask some more about the model. The width of the brush in contact with the floor obviously scales as h. To maintain strict proportionality the front-to-back length of the brush in contact with the floor should also scale as h. But a better model for “constant cleaning power” would be for the front-to-back length to be determined by the interaction of the brush with the floor, independent of the brush width. Constant cleaning power would also require that the rotational speed of the brush against the floor be independent of the scale of the machine. Under these assumptions P would scale as h, and the running time would be proportional to h^2. Now scaling down would be *really* costly: the 30 cm diameter machine that ran for 30 minutes, when scaled down to 1 cm, would run for only 2 seconds.

3.5 Constant Cleaning Power

Note that “constant cleaning power” was defined in terms of having a constant front-to-back length of brush in contact with the floor and a constant rotational speed of the brush with respect to the floor. But what about the forward velocity of the machine over the floor? Again, strict proportionality would say the forward velocity should scale as h, but for the machine to be really useful it is probably more realistic for the forward velocity, like the front-to-back length of the brush in contact with the floor, to be independent of the scale of the machine. The area of floor cleaned per unit time would then depend only on the width of the brush, which scales as h. Since the running time scales as h^2, the area cleaned in the machine’s running time scales as h^3. The 30 cm diameter machine scaled down to 1 cm would clean only $(1/30)^3$ of the floor area before running its batteries down. This might not actually be as bad as it sounds, inasmuch as an alternative reasonable expectation might be that a machine 1/30 as wide would clean only 1/30 as much floor area, in which case the smaller machine would fall short of our expectation by only a factor of $(1/30)^2$ versus $(1/30)^3$.

3.6 Energy Lost to Body Drag

We could go on for a long time examining different scenarios and assumptions, but let’s do just one more. Let’s assume the machine picks up dirt in some undisclosed but very efficient way that consumes practically no power. The power cost of using the machine is then its frictional drag across the floor and through the air. Both frictional costs scale, to a good approximation, as the product of the area and velocity, h^2 v. The machine running time thus scales as h/v. If we assume a constant-velocity-over-the-floor model then the running time still scales as h, so the 30 cm / 30 minute machine scales down to a 1 cm / 1 minute machine. But if we take another alternative reasonable assumption, one that says our expectation is for a the smaller machine to move across the floor more slowly in proportion to its diameter, i.e., v is proportional to h, then the running time is independent of scale: all members of this family of vacuum cleaners run for 30 minutes. However the area cleaned in running time t scales as h v t, so with v proportional to h the area cleaned scales as h^2, still falling short of our expectation that the area cleaned in the machine’s running time might reasonable scale as h.

3.7 Total Cleaning Power

Finally, we can start with the goal of building a family of machines each of which cleans an area proportional to its diameter in whatever its running time may be and ask what is the corresponding power consumption model. The area cleaned is h v t, and we will be satisfied if it is proportional to h, i.e., if v t – equal to the linear range of the machine – is constant. From our very first analysis we have t proportional to h^3/P, so we require h^3 v / P to be constant, or the power consumed to be proportional to h^3 v. Since the machine's mass is proportional to h^3, this is exactly the power cost of hill climbing: a robotic vacuum cleaner most of whose energy is spent on going uphill at constant speed will clean an area proportional to its linear dimension, will traverse an altitude change independent of its scale, and will do it in time that is independent of scale and (obviously) inversely proportional to speed, which may be chosen arbitrary. This result is similar to but not exactly the same as the "all geometrically similar animals jump to the same height" observation, inasmuch as in the running up hill case we have specified that the velocity is arbitrary but constant, whereas in the jumping case the velocity is linear in the time.

3.8 Best General Answer

There is no best answer. There is not even a single answer, because all of the models discuss are to some extent simultaneously realized in every device; the real question for any particular device is what is the relative weighting of these and other energy loss mechanisms. For a mobile robot whose main job is to provide remote human observers with sensor information obtained by sensors mounted on the robot the energy requirement is likely to consist of a constant component related to information processing and communication, an h^2 component related to maintaining a suitable operating temperature, and an h^2v component relating to viscous drag. The last may be the most interesting, as on the practical side it will be the dominant term for high performance high speed robots, and on the theoretical side it leads to an interesting invariance worth keeping in mind.

This invariance is derived in Section 3.6, but it bears repeating and a high-level interpretation here. The model is that the dominant energy loss term is viscous drag, power proportional to the product of frontal area and velocity – the mechanical equivalent of Ohm's Law. With P thus proportional to h^2 v and carried energy proportional to h^3 we obtain running time t proportional to h/v. It is useful to think of h/v, the time it takes to robot to move one body length, as a step time; robot running time *measured in step times* is thus independent of robot scale, and robot range *measured in steps* is also independent of robot scale, with the same proportionality factor.

4 Conclusion

After setting up the background context so as to give the reader a concrete scenario in which a variety of performance expectations and scaling issues could be considered, discussion focused on a hypothetical family of geometrically similar robotic vacuum cleaners. No doubt the reader will appreciate the underlying universality of the principles and the approach, and with this appreciation be able to pose and answer questions about the range, running time, and a variety of other performance considerations for mobile robots in general. For mobile robots of characteristic dimension h and velocity v in which the dominant energy loss mechanism is drag, if we think of h as a step length and h/v as a step time, the range in steps and the running time in step times are both independent of robot scale. This is probably the most realistic single-term model for modern vehicles, e.g., automobiles, aircraft, and ships. By comparison current generation mobile robots are over designed and underperforming; it is nevertheless entirely reasonable to expect that what is now the best model for high performance transportation will in the future also be the best model for high performance robots.

5 Acknowledgements

Some of the analysis of scaling issues related to the problem of communications within and with a large network of small sensors was stimulated by discussions with John Manobianco of ENSCO.

6 References

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