Multicommodity Flow Models, Failure Propagation, and Reliable Loss Network Design

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Abstract—Multicommodity flow (MF) models are well known and have been widely used in the design of packet-switched networks. They have also been used as approximations in the design of circuit-switched networks with reliability constraints. In this paper, we investigate the usefulness of multicommodity models both as routing models and as an integral part of design models conceived under the failure propagation strategy. First, we compare the performance measures calculated by the models with results produced by a real-time technique. Next, we study the performance of networks dimensioned with flow models and with known adaptive models under failures of transmission facilities when a real-time routing technique is used. Results obtained using realistic data show that the MF models compare favorably with exact dimensioning algorithms when failures are considered.

Index Terms—Adaptive routing, circuit switching, integrated design, network design, reliability.

I. INTRODUCTION

THE NOTION of reliability has gained great importance in recent years. The design of transmission networks under reliability constraints is a complex nonlinear optimization problem that typically requires large amounts of computation time to produce an optimal solution. It is often solved by heuristics [1], [2], [7], [23], [24], [33] based on the classical network synthesis problem. It uses the well-known theory of multicommodity flows (MF’s) for which there is a large body of work, both theoretical and numerical, and can produce fast solution algorithms [28]. The MF model has also been used as the basis for a design algorithm [11], [13], [20], [21] of circuit-switched networks. This technique has been extended to the reliable design of switched networks in [35] where a reasonably fast algorithm has been proposed for the synthesis of telephone networks with reliability constraints.

The efficiency of these design methods when used for switched networks comes at the cost of some strong simplifying assumptions. Most important are: 1) the representation of the traffic blocked on a path as the sum of blocked traffic on all the links on that path and 2) the conservation of flow at all nodes in the network. Another difficulty with the MF model lies with the real-time implementation of a routing policy. The flow model yields what can be interpreted as average carried traffic on the paths that are used in the solution. In practice, a network operator needs a rule that states, for each call arrival, whether this call should be accepted and, if so, on which path it should be connected. Given an MF, the construction of a rule such that the real-time carried traffic will be identical to the flows can be quite difficult [8] and the resulting algorithm will generally be far from optimal.

In the traditional literature on network reliability and network synthesis, the notion of failure has been considered either at the physical level or at the logical level. By physical level we mean what is also known as the transmission or facilities network in the telecommunication literature. Similarly, the logical level corresponds to the switched network. The term failure propagation has only been recently coined in the literature for asynchronous transfer mode (ATM) networks [30] and for the propagation of signaling errors to switched networks [17]. The idea that failures actually propagate from the physical network to the logical level was well known in the industry but was rarely addressed in a direct fashion in mathematical models. Recently, models have been proposed [9], [27] to assess the performance of teletraffic networks taking into account the effect of failure propagation.

The objective of this paper is twofold. First we want to examine more closely the relationship between the flow solution and real-time routing in the context of network dimensioning with reliability constraints. We also want to investigate the benefits obtained by taking into account the reliability and failure propagation constraints in the dimensioning process. For the first objective, we present in Section II the MF model applied to the logical network, its solution, and a method to evaluate an end-to-end blocking probabilities. The real-time routing algorithm (DCR) is briefly explained in Section III. Section IV shows the comparison between the two routing algorithms. We begin to address the second objective in Section V where a synthesis model that accounts for failure propagation and uses the multicommodity model is proposed. Numerical results are discussed in Section VI.

II. THE MF MODEL

Let the switching network be represented by the capacitated graph $G(N, M)$ where $N$ is the set of switches and $M$ the set of logical links, also known as trunk groups, between them. Let $R$ represent the set of all origin–destination (O–D) pairs in this graph. For every O–D pair $k \in R$, there is a demand $d_k$ which is the arrival rate of call requests for this pair.
These call demands will compete for the capacity available. In the classical cooperative model, also called the system-optimal model, the network is operated in such a way that a suitable cost function is globally minimized subject to capacity constraints. This cost function may be a real cost or, in the case where the network is given, a performance measure that we would like to optimize.

In the case of packet-switched networks, such a function is the total expected delay of packets across the network [19]. This function is a reasonable objective to minimize when the network structure and dimensions are known and fixed. Because packet-switched networks guarantee the conservation of packets in transit under normal conditions, the classical MF with a nonlinear objective and capacity constraints is a natural model to use for the design of these networks. Using the expected delay as objective has the added advantage that it acts as a natural penalty function when the flows approach the values of the link capacities. The capacitated MF model can then be transformed into an uncapacitated version, thereby simplifying the solution algorithm [12]. Moreover, if all of the O–D delays are equally penalized, the objective function is convex and the solution is a global minimum.

This is not to say, however, that the MF model can be used blindly for reliability of packet networks. Many other phenomena occur when failures occur, especially packet loss and transient regimes that are not well modeled by conservative flows. Our point here is simply that MF models have become quite popular in telecommunication network design because of their success in the design of packet networks where for which they are particularly well suited.

For circuit-switched networks, the usual objective function used in the optimization of routing is the total end-to-end loss probability, or equivalently, the total lost traffic. In the case where a specific routing technique is selected, the optimization is generally done by specialized models different from the MF model. If the routing algorithm is not specified, it is possible to define an MF in the network that represents the time-average number of calls on various paths.

Nevertheless, using the end-to-end loss probability as the objective function is not very well suited to this approach.

1) The form of this function depends strongly on the real-time routing method used in the network.
2) It is not separable by link and, as a consequence, the standard solution technique of [12] is not directly applicable.
3) It does not contain a natural penalty term so that the capacity constraints still have to be explicitly taken into account.
4) It is not convex and the solution is only a local minimum.

Because MF models are well known, they have been proposed in [37] as an approximate routing model for circuit-switched networks. The end-to-end loss probability is approximated by an objective function that does not suffer from the defects mentioned above and is thus better suited to the multicommodity model. The approximation is to take the sum of the blocked traffic on all links of the network as an estimate of the total lost traffic. With this approximation, it can be shown that we obtain a standard MF model with all the desired properties.

Let \( O \) be the set of all possible origin switching nodes and \( X_p^q \) the expected number of call attempts originating at \( p \in O \) on link \( u \). In a network where the link blocking probabilities are small, \( X_p^q \) is also the expected number of calls present on the link. In this case, these flow variables satisfy the usual conservation constraints at node \( i \)

\[
\sum_{u \in \Gamma^+(i)} X_u^i - \sum_{u \in \Gamma^-(i)} X_u^i = \begin{cases} 
F^{pq}, & \text{if } i = p \\
-F^{iq}, & \text{if } i = q \\
0, & \text{otherwise}
\end{cases} \quad (1)
\]

and define an MF, each commodity corresponding to a particular origin \( p \). In (1) and (2), \( \Gamma^+(i) \) and \( \Gamma^-(i) \) are the sets of arcs into and out of node \( i \) and \( F^{pq} \) is the demand from node \( p \) to node \( q \). It represents the average number of calls trying to enter the network at node \( p \) for destination node \( q \). The routing optimization problem is then to minimize a suitable objective function \( z(X_p^q) \) subject to these constraints. As we said before, a natural choice for this function is the total lost traffic or total loss probability. Because the exact function is not well suited to the multicommodity approach, it has been approximated by [37]

\[
z \approx \sum_{u \in M} P_u(X_u^i, C_u) \quad (3)
\]

\[
P_u(X_u^i, C_u) = X_u^i E(X_u^i, C_u) \quad (4)
\]

\[
X_u^i = \sum_{p \in O} X_p^i \quad (5)
\]

\[
E(X, C) = \frac{X^C}{\sum_{i=0}^{C} X^i / i^!} \quad (6)
\]

where

- \( M \) total number of links in the switched network;
- \( P_u \) total traffic blocked on link \( u \);
- \( X_u^i \) total traffic offered to link \( u \);
- \( C_u \) capacity of the link;
- \( E(A, C) \) Erlang-B function.

This model introduces two approximations which appear at first sight to contradict each other. First, it assumes conservation of the flow variables. This is not unrealistic when the link blocking probabilities are small and without this assumption, a generalized MF with nonlinear gains should be used, for which there does not seem to be an efficient solution technique. This is why the \( X_u^i \) variables represent in our model the offered and carried traffic, since we do not distinguish between these two quantities.

Even though flow conservation is assumed, there must be some way to represent the fact that some traffic will be lost. The second approximation is to consider that the objective function is separable by arc and that the lost traffic is the sum of the blocked traffic on all the links in a path. This of course will overestimate the amount of lost traffic although it is not known by how much without actually doing the calculation. This approximation should get better as the link blocking probabilities get smaller, and also it should be better for shorter paths than for longer ones.
A. Solution of the Routing Model

The routing model defined above is a convex nonlinear uncapacitated MF. The convexity of the objective function is guaranteed by the convexity of the $P_d(X_u,C)$ function, the total traffic blocked on link $u$ (see [31], a different proof can be found in [34]).

The convexity of the objective guarantees that a global optimum can be found by an efficient descent algorithm. A convex simplex implementation was proposed in [36], but in this paper we use the classical Frank–Wolfe method also known as the flow deviation algorithm [12]. The steps of the flow deviation method will be used to record information that will be used later in the evaluation of a more detailed performance measure used in the comparison with the real-time routing technique. For this reason, we summarize the algorithm. At iteration $n$, $V^{(n)}$ represents the set of shortest paths between all O–D pairs where the link length vector is $\nabla z$ with respect to the flow variables; $\lambda^{(n)}$ is the optimal step size in that direction, in the sense of minimizing

$$z[(1-\lambda^{(n)})X^{(n)} + \lambda^{(n)}V^{(n)}], \quad 0 \leq \lambda \leq 1$$

and $\epsilon$ is the tolerance for stopping. Then the algorithm can be summarized as follows:

1) find a feasible starting solution $X^{(0)}$. This can be done by using the flow deviation algorithm itself with a reduced demand, as described in [12];
2) let $n = 0$;
3) at iteration $n$, compute the new flow

$$X^{(n+1)} = (1-\lambda^{(n)})X^{(n)} + \lambda^{(n)}V^{(n)}$$

4) stopping rule (several stopping criteria can be found in [25] and [3]). If the stopping rule is not respected, go to step 3.

B. End-to-End Loss Probabilities

The MF model was defined so that an approximate measure of the total lost traffic is minimized. In practice, a more precise measure of the grade of service than this is often required by the planners, who need to know the loss probability of individual O–D pairs. The calculation of these individual loss probabilities is difficult since the flow model is stated in terms of link flows. Since there are no prespecified paths from which to choose, the mapping of a set of link flows into a set of path flows is not unique and has to be computed from the link flow solution.

One possibility would be to restate the model in terms of path flows, something that is theoretically easy to do. The problem is that implementation quickly becomes unmanageable as the size of the networks, and the number of paths, increases. Instead, we present in this section a method to compute the end-to-end loss probabilities directly from the Frank–Wolfe algorithm.

First we have to estimate which paths are used by each O–D pair and how much of the total flow of this pair goes on each path. This is expressed by the notion of a path selection probability $Q_k^{ij}$, which is the fraction of the $(i,j)$ traffic that is routed on the $k$th path between nodes $i$ and $j$. If $L_k^{ij}$ is the end-to-end loss probability for $(i,j)$ calls and $L_k^{ij}$ is the blocking probability on path $k$, then we have

$$L_k^{ij} = \sum_k Q_k^{ij} L_k^{ij}.$$ 

Note that this formulation assumes that a call that is blocked on a path is lost; in other words, there is no alternate routing of calls. It also assumes that the selection probabilities are independent of the link blocking probabilities, which may or may not be the case in practice, but is certainly not the case for the flow model. In any event, the selection probability is given by

$$Q_k^{ij} = \frac{x_k^{ij}}{F^{ij}}$$

where $F^{ij}$ is the total demand from $i$ to $j$ and $x_k^{ij}$ is the traffic carried on the $k$th path between $i$ and $j$.

Assume that path $k$ is composed of links $u_1,u_2,\ldots,u_r$ and that the blocking probability of link $u$ is given by $B_u$. A simple expression for the path blocking probability, assuming link independence, is given by the reduced-load model

$$L_k^{ij} = 1 - \prod_{u=1}^r (1 - B_u),$$

This is often replaced by the so-called flow model

$$L_k^{ij} = B_1 + (1 - B_1)B_2 + (1 - B_1)(1 - B_2)B_3 + \cdots (1 - B_1)(1 - B_2)\cdots(1 - B_{r-1})B_r$$

which is equivalent to the reduced-load model for low values of the link blocking probabilities.

The end-to-end loss probability $L_k^{ij}$ can thus be calculated if we can find the path flows $x_k^{ij}$. For this, we have to make use of the way the Frank–Wolfe method operates. In the Frank–Wolfe algorithm, the paths are not chosen beforehand but rather the flows are assigned to paths at each iteration. At the first iteration, all of the $(i,j)$ flow is sent on the shortest path. At the second iteration, part of this flow will be moved to another path. This process can be summarized as follows for a given $(i,j)$ pair, where $x_k^{(n)}$ is the flow on the $k$th path of pair $(i,j)$ at iteration $n$. At iteration 0, we have, dropping the $i,j$ indices to simplify notation

$$x_0^{(0)} = F$$

at iteration 1

$$x_0^{(1)} = (1 - \lambda^{(0)})F$$
$$x_1^{(1)} = \lambda^{(0)}F$$

at iteration 2

$$x_0^{(2)} = (1 - \lambda^{(0)}) (1 - \lambda^{(1)})F$$
$$x_1^{(2)} = (1 - \lambda^{(1)}) \lambda^{(0)}F$$
$$x_2^{(2)} = \lambda^{(1)}F$$
and, in general, at iteration \( n \)
\[
x_k^{(n)} = \begin{cases} 
\prod_{j=k}^{n-1} (1 - \lambda^{(j)}) F_j, & \text{if } k = 0 \\
\lambda^{(k-1)} \prod_{j=k}^{n-1} (1 - \lambda^{(j)}) F_j, & \text{if } 0 < k < n \\
\lambda^{(k-1)} F_k, & \text{if } k = n.
\end{cases}
\]
From this, we compute the selection probabilities by (7). It is then a straightforward matter to keep track of the amount of flow that is actually rerouted during the flow deviation algorithm and to reconstruct the final path flows and the end-to-end loss probabilities.

III. A REAL-TIME ROUTING: THE DCR METHOD

The dynamically controlled routing (DCR) has been described in [6] and is currently implemented in the Stentor network in Canada as well as in other telephone networks. It is an adaptive method that can react to changing network conditions and for this reason is very relevant to the reliability aspect of network operations. The performance evaluation of a network operating with DCR has been described in [15] and [32], and dimensioning algorithms based on this method are described in [16] and [18]. In what follows, only the most important features of the DCR method are summarized. The interested reader may refer to [6] and [39] for more details.

New calls are first offered to the direct link between the origin and destination offices. If the call is blocked, it is offered to a single two-link alternate path. If the call is blocked on this path, it is lost. The alternate path selection can be deterministic or probabilistic. In the probabilistic formulation, the probability \( P^{ri,j}_{k} \) of selecting a tandem node \( k \) for an \((i,j)\) call is computed at fixed interval and depends on \( \tilde{r}^{ri,j}(t + \tau) \), the estimated residual capacities of the path links at time \( \tau \) from time \( t \). It is given by
\[
P^{ri,j}_{k} = \frac{\min \{ \tilde{r}^{ri,k} + \tilde{r}^{ri,j} \} \min \{ \tilde{r}^{ri,j} + \tilde{r}^{ri,m} \}}{\sum_{m} \min \{ \tilde{r}^{ri,m} \}}.
\]

The number of free trunks on a link \((i,j)\) at time \( \tau \) from now, also called the residual capacity, is estimated by the linear extrapolation
\[
\tilde{r}(t + \tau) = \max \{ 0, C - \eta(t) - \tau [Y(t) - \eta(t)/S] \}
\]
where the \((i,j)\) subscripts have been dropped for simplicity, and
\[
\eta(t) \quad \text{number of busy trunks at the current time;} \\
Y(t) \quad \text{moving average of the call arrival rate on the link;} \\
S \quad \text{average call holding time;} \\
C \quad \text{number of trunks on the group.}
\]
The sliding window estimation of the call arrival rate is given by
\[
Y(t) = \theta Y(t - \Delta) + (1 - \theta) M/\Delta, \quad 0 \leq \theta \leq 1
\]
where \( \Delta \) is the so-called update cycle time and \( M \) is the actual number of calls offered to the group during the last interval \( \Delta \). The parameter \( \theta \) is a weight assigned to the previous values and is a fixed parameter of the algorithm that is selected \textit{a priori}. This version of the DCR routing algorithm does not have state protection. While state protection is easily added to the DCR dimensioning tools, this is much more difficult to do for the multicommodity model and for this reason, it has been left for further work.

In this paper, we will be using the method of [16] for the performance evaluation and the dimensioning. This method (that will be later referred as "DCRsyn") uses a static version of the real-time algorithm since dimensioning is generally done for the long-term expected value of the traffic. For this reason, the parameters \( \Delta, \theta, \) and \( S \) do not appear in the dimensioning model.

It has been often stated [6], [5], [39], [4], [10], [38], sometimes with little evidence, that the adaptability of the routing can take care of failures. In that sense, it is not surprising that none of the DCR algorithms found in the literature has been designed taking failures into account. Unfortunately, this adaptation to failures is not generally very good, as shown in [29] where a detailed comparison was made between an adaptive and a fixed alternate routing technique. In the presence of link failures it was shown that the adaptive method did not recover as much traffic as the optimal fixed sequence, thus indicating the need to further optimize the network to take into account the reliability requirements. In fact, some of the results of this paper will confirm these results.

IV. COMPARISON OF FLOW PATTERNS

The first set of results concerns the realization of a given MF by the DCR algorithm. In other words, how does the computed flow compare with the traffic pattern of the DCR routing in a given network? A simple measure of the efficiency of a routing technique is the total lost traffic. Although this is an important parameter, we think that it is not sufficient to guarantee adequate performance in actual operations. Moreover, the DCR algorithm has not been designed to optimize a specific objective function. Thus, there is no reason why the value of the carried traffic obtained by the DCR algorithm should be equal to the value obtained by another method based on optimizing the carried traffic. What we know [14], on the other hand, is that if the objective is to maximize the carried traffic, then it is the marginal weighted revenue of all paths that has to be equalized, not the path residual capacity, as is done in DCR. It is also conceivable that two routing techniques may have the same total lost traffic but very different grade of service measures for particular \(O-D\) pairs. We want to ensure that the grade of service is met for all users of the network individually and this is why we compare the individual end-to-end loss probabilities for all \(O-D\) pairs for the two performance evaluation algorithms.

The complete representation of an MF or of the flow pattern of the DCR algorithm becomes very rapidly unmanageable as the size of the network increases. For this reason, we have chosen to compare instead the end-to-end loss probability for each \(O-D\) pair produced by the two techniques. This is an important quantity for network performance since it is the main measure of the grade of service seen by the user and is also a measure of the lost traffic.

First, a network was dimensioned with the multicommodity model presented in [35] and the end-to-end loss probabilities...
were estimated based on the method of Section II-B. The endto-end loss probabilities were then recalculated when the same network was operated with the DCR routing. This was done by the performance evaluation algorithm that is an intrinsic part of the dimensioning algorithm described in [16]. A typical result is shown in Fig. 1 for a 17-node network based on real traffic values. It shows histograms of the end-to-end loss probabilities for all o-d pairs. It is quite clear that the loss probabilities, and hence the traffic patterns, are significantly different for the two methods; the multicommodity model has a wider spread of loss probabilities that the actual DCR routing.

A similar comparison of the value of the total loss probability calculated by the two models is made on Fig. 2 in the presence of failures. For that figure, we have assumed that all the circuits of a given trunk group have failed. The histogram shows only the flows that have a loss below 0.1. For both algorithms, only two flows have a loss above this value. In each case, this is the flow whose direct link is the failed trunk group. For the DCR algorithm, this loss probability is estimated at 0.29, while the multicommodity model gives a value of 0.12. Here again we see that the distribution of end-to-end loss probability produced by the DCR algorithm is quite different from the values calculated by the multicommodity model which often gives a spread of values wider than what is actually produced by the DCR routing.

The obvious conclusion is that the flow patterns predicted by an MF model differ substantially from the patterns generated by the DCR routing. One of the reasons that could explain such differences is that MF does not restrict the number of links used in the call routing whereas DCR does. We can also expect that this will be so for other types of nonhierarchical routing techniques, although this, of course, remains to be verified.

V. FAILURE PROPAGATION AND DIMENSIONING

Even though the traffic model may not be very accurate, we may nevertheless want to use it to dimension a network since numerical algorithms for flow models are very efficient. Can we then expect that the resulting network will have a reasonable grade of service under failure when it is used with a real-time routing algorithms such as DCR? And if so, how much can we gain by modeling the reliability requirements over a straightforward dimensioning technique with an adaptive routing?

Another related question is how to integrate the failure propagation phenomenon into the synthesis procedure. Thus, before entering into the performance details of the two techniques, we need to further discuss failure propagation and dimensioning. Up to this point, the physical layer of the network and its relationship with the logical level has not been considered. For this, additional notation is need. Let $G^p(N^p, M^p)$ be the graph representing the physical network, $N^p$ the set of nodes, and $M^p$ the set of links. Recall that in the transmission networks, nodes are multiplexers and links are transmission systems. Note that all switches are co-located with a transmission multiplexer although in many cases, there can be more switching nodes than transmission nodes.

The relationship between $G^p$ and $G$ is described as follows. Let $u = (i, j) \in M$ be a link of the logical network that originates at switching node $i \in N$ and terminates at switching node $j \in N$. The capacity of this link $C_{ij}$ is seen at the physical level as a demand of circuits that must be physically routed through the multiplexers and transmission systems. The circuit routing problem is then to find the amount of circuits $y_{jk}^u$ that will be routed on each path $k$ between $i$ and $j$. This is an entirely different problem from the call routing problem in the switched network that we are considering in this paper. Nevertheless, the actual circuit routing must be somehow included in the model since it affects the performance of the switched network.

This can be seen from Fig. 3. The trunk group between nodes $A$ and $B$ has 50 circuits which are routed on two paths in the transmission network: 25 circuits on path $(A, E, B)$ and 25 circuits on path $(A, F, D, B)$. Similarly, the trunk group between nodes $C$ and $B$ has 80 circuits, which are routed on two different paths in the transmission network: 50 circuits on path $(C, E, B)$ and 30 circuits on path $(C, F, B)$. Suppose now that the transmission link $(E, B)$ is cut off. Paths $(A, E, B)$ and $(C, E, B)$ then become unavailable so that both trunk groups now appear to have reduced sizes, trunk group $(A, B)$ being reduced to 25 circuits and trunk group $(C, B)$ to 30 circuits.

For the sake of simplicity, we have assumed here that the circuit routing is given and that each trunk group is equally routed in graph $G^p(M^p, N^p)$ through the first and second arc-disjoint physical shortest paths between pairs $i \in N^p$ and $j \in N^p$. This corresponds to the standard practice of diversity routing in transmission networks and requires that the network
be two-connected. It means that for any single link failure, all trunk groups will have at least 50% of their capacity available. It also means that the failure of a single transmission link may affect more than one trunk group, depending on the circuit routing.

The reader should be aware, however, that the 50–50 routing requirements are not essential to the modeling approach. In Section VI-F, we show results for which the circuit routing percentage is changed.

We define the state of the transmission network by the set of its failed links. We assume that a link is either fully functional or completely out of service; there are no partial failures. The total number of states is then given by $2^{|M_F|}$. Let $S_k$ be a particular state of the transmission network. Each state has a probability $Pr(S_k)$ and a performance measure $z(S_k)$ associated to it. An overall network performance measure $E$ is then defined as

$$E[z] = \sum_{k=0}^{2^{|M_F|}} Pr(S_k)z(S_k)$$  \hspace{1cm} (9)$$

where by convention $S_0$ denotes the state with no failed link. The expression (9) is completely general and cannot be evaluated in reasonable time for realistic networks. It can be simplified if we assume that, among all of the $2^{|M_F|}$, only $t$ states are significant. In that case, bounds on an expected performance measure can be found [22], [36] by considering only the contribution of those $t$ states and we get

$$E[z] \approx \sum_{k=1}^{t} Pr(S_k)z(S_k)$$  \hspace{1cm} (10)$$

as an approximation. In this paper, the $t$ significant states are single failures of transmission links. The probability of being in a state $S_k$ can be found as follows. Let $p_l$ be the availability of transmission link $l$, $q_l = 1 - p_l$ be the unavailability of the link, and $T_l(S_k)$ be

$$T_l(S_k) = \begin{cases} 0, & \text{if link } l \text{ is available for state } S_k \\ 1, & \text{otherwise} \end{cases}$$

Then

$$Pr(S_k) = \prod_{l \in M_F} p_l^{T_l(S_k)}$$  \hspace{1cm} (11)$$

In the equation above, the link unavailability is evaluated as the proportion of the mean down time per unit of length (in hours per 100 km) over the total operating time multiplied by the link length. Concerning the performance measure, $z(S_k)$ is chosen depending on the definition of reliability that is used. In our case, it should reflect the relationship between the physical network and the call routing performance and it is given by the expected lost traffic in state $S_k$. The connection can be made explicit by noting that, in general, when there is no restriction on the paths used in the transmission network, the trunk group sizes $C_{i,j}$ in the switched network are related to the circuit routing variables $y_{m}^{i,j}$ by

$$C_{i,j}(S_k) = \sum_m y_{m}^{i,j} \delta_{m}^{i,j}(S_k)$$

where

$$\delta_{m}^{i,j}(S_k) = \begin{cases} 1, & \text{if path } (i,j,m) \text{ is available in state } S_k \\ 0, & \text{otherwise} \end{cases}$$

and $y_{m}^{i,j}$ is the number of circuits of trunk group $(i,j)$ that are routed on the $m$th path between these nodes in the transmission network, denoted here as $(i,j,m)$. When a transmission link fails, some of those paths become unavailable, the trunk group sizes are reduced, and the switched network performance is degraded. We then have

$$z(S_k) = \sum_{u \in M} P_u(X_u, C_u(S_k))$$

which is simply (3) written for the set of trunk group sizes corresponding to state $S_k$.

Let $K(C)$ be a function of the vector of logical link capacities $C$ that represents the cost of dimensioning the logical network. A network synthesis model with reliability and failure propagation constraints can be seen as the problem of finding the set of flow and capacity variables such that there is a suitable tradeoff between the cost of capacity and the costs of lost call traffic in the event of failures. We must now solve the dimensioning problem

$$\min_{S_k \neq C} K(C) + u_0 Pr(S_0)z(S_0) + u_1 \sum_{k=1}^{t} Pr(S_k)z(S_k)$$  \hspace{1cm} (12)$$

subject to

$$\sum_{u \in \Gamma^+(i)} X_{u}^p - \sum_{u \in \Gamma^-(i)} X_{u}^p = \begin{cases} F^{pq}, & \text{if } i = p \\ -F^{pq}, & \text{if } i = q \\ 0, & \text{otherwise} \end{cases}$$

$$X_{u}^p \geq 0.$$
In this model, the variables $C_{i,j}$ represent the trunk group sizes that have to be determined. As such, they are modified by the optimization procedure. The objective function is evaluated by taking into account routing diversity as follows. The set of paths that can be used for each $(i,j)$ pair and the fraction of the circuits of trunk group $(i,j)$ that is routed on each path are given and do not change during the optimization. Suppose that at a point in the optimization procedure we have the current value of a trunk group size $C_{i,j}$. For each state $S_k$, we know which of these paths are available and we can compute directly the value of $C_{i,j}(S_k)$. For example, in Fig. 3, we have $C_{A,B}(S_0) = 50$. In state $S_1$, where transmission link $(E,B)$ has failed, we know that 50% of these circuits will not be available since we have assumed that the circuit routing is equally split between the two paths. We then immediately know that in state $S_1$ the apparent size of trunk group $(A,B)$ is only $C_{A,B}(S_1) = 25$. This argument is valid for all trunk groups, all fixed sets of paths, and all fixed partitions of trunk group circuits among these paths.

In (12), multipliers $u_1$ and $u_2$ can be interpreted as penalty terms for the lost call traffic or as the Kuhn–Tucker multipliers of a dualized grade of service constraints. They can be set manually to control the loss under failure scenarios, as was done in this paper, or they could be computed by a subgradient procedure. We want to emphasize that the values of $u_1$ and $u_2$ are not chosen arbitrarily. Rather, they are dictated by the requirement that the grade of service constraints should be met at the solution. The actual values will vary from problem to problem depending on the actual traffic and cost data. The actual values are only interesting as an estimate of the cost of lost traffic, something that is all too often simply chosen arbitrarily in many design algorithms.

Note that the no-failure and the failure states were separated in order to provide more flexibility in terms of grade of service. In this way, the planner has the possibility of adjusting the multipliers to penalize more or less the failure states with respect to the normal state.

In what follows, we give the name “MFsyn” to the synthesis procedure based on (12), (1), and (2) to clearly distinguish it from the synthesis procedure based on DCR, called “DCRsyn.” The resulting networks will be called the MF network and the DCR network, respectively.

The resolution approach for MFsyn is the one described in [35] and is a variant of the coordinate search method. It consists of decomposing the problem into two major subproblems: an MF problem with fixed capacity variables and a link dimensioning problem for which the flow variables are fixed. The algorithm starts with a set of capacities, and, one by one, all of the MF problems for each physical failure state have to be solved. The link flow variables are then fixed and the search is carried out for the set of link capacities that minimizes (12).

The convergence of such a procedure to a global minimum can be guaranteed only if the objective function is jointly convex on the $X$ and $C$ variables. Unfortunately, the objective function (12) is composed of a linear function $K(C)$ and a function of the lost call traffic. The lost call traffic is convex over variable $X$ and variable $C$ but the joint convexity has, to our knowledge, not been proven.

### VI. Results

The network used for comparative purposes is depicted in Fig. 4. This network, as well as the demand matrix, are real data that have been used in [26]. In the figure, switching nodes are depicted by a circle whereas transmission nodes are represented by squares. The computational experiment was designed as follows.

1) Dimension the logical networks using the MFsyn and the DCRsyn methods described earlier. Compare the computational requirements of the two techniques and the cost of the resulting networks.
2) Evaluate the performance of the two networks when the DCR routing algorithm is used on both networks for call routing.
3) Consider single physical failures and study the impact of those failures in both networks under DCR call routing.

#### A. Comparison of the DCRsyn and MFsyn Synthesis Methods

The first comparison results are provided in Table I where the dimensioning cost, the computational time, and the grade of service for the no-failure scenario is presented for the two synthesis methods in the case of no failure. In that table, the total loss probability is the fraction of the total traffic offered to the network that is lost. It is approximated by the penalty term of the objective function (3). The costs presented in the table are the trunk group dimensioning costs $K(C)$ and do not include the penalty costs. They are evaluated using the set of cost characteristics and distances provided in [26]: a termination cost of $300 per port and an airline distance cost of $0.05 per mile. The first of the two traffic matrices presented in [26] was used for this network. As we said before, the penalties $u_1$ and $u_2$ were adjusted to provide a suitable grade of service of 1% in the normal state and 20% in any failure state.

Typical values of $u_1$ and $u_2$ are shown in Table II for three different circuit routings: an equal partition of the circuit demand on the first and second shortest paths, then a 67% allocation of the first shortest path and the rest on the second shortest path, and, in the third case, the reverse values. The large difference in the values of the multipliers indicates that the objective function is more sensitive to the penalty.

![Table I: Comparison between DCRsyn and MFsyn Network Design Algorithms, No Failure](image-url)
terms than that of the constraints under normal operation, even though the value of the grade of service constraint is more stringent. This is probably due to the fact that all constraints in the failed state are aggregated, but it would be an interesting subject of further research to see more precisely which reliability constraints have the largest impact on the objective function. Also, these values would obviously be different for different networks. All of the results presented in this section were obtained for a 50–50 circuit routing; results for the other two routings are summarized in Section VI-F.

As can be seen from Table I, both the DCRsyn and MFsyn networks are comparable in terms of costs and average performance. The fact that the MF network is slightly more expensive than the DCR network is expected, given that the MF dimensioning algorithm takes explicitly into account all of the single physical link failures in the procedure. Therefore, we expect the MF network to be slightly overdimensioned with respect to DCR network.

The actual trunk group sizes are shown in Fig. 5 and the difference in dimensioning in Fig. 6. The group sizes produced by the two methods are somewhat different but, as we will see later, these small differences in trunk group sizes have a significant effect on the network behavior under failure. In terms of computational time, on the other hand, the MF synthesis algorithm is much faster than the DCR method. In fact, the MF algorithm takes almost half the time to dimension the network even though it takes into account all failure states in the optimization process, while the DCR-based algorithm has no such reliability requirement.

### B. Computation Time

The main reason for using the MF model, with all of its simplifications, is given in Table III. A set of networks were randomly produced. This table shows the growth of the computation time for the multicommodity and the DCR algorithms as a function of network size, expressed as the number of nodes. It is quite clear that the computation requirement of the DCR method grows much faster than that of the MF algorithm. In fact, we could run the MF algorithm on networks of up to 100 nodes (switching and transmission each) in about two hours. Extrapolating the values of Table III, we estimated that the DCR algorithm on that problems would take about five days of CPU time and we decided that we did not want to dimension networks above 65 nodes.

### C. Loss Probability Under Failure

The second step of our comparative procedure is the evaluation of network performance in the failure states. We want to

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Table II

<table>
<thead>
<tr>
<th>% Allocation to 1st–2nd shortest path</th>
<th>$w_0$</th>
<th>$w_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50–60</td>
<td>200</td>
<td>90000</td>
</tr>
<tr>
<td>67–33</td>
<td>200</td>
<td>73000</td>
</tr>
<tr>
<td>33–67</td>
<td>200</td>
<td>78000</td>
</tr>
</tbody>
</table>

---

Fig. 4. Network used for numerical results.

Fig. 5. Trunk group dimensioning by the MF and the DCR algorithms.

Fig. 6. Difference in trunk group sizes: MF dimensioning–DCR dimensioning.
TABLE III

<table>
<thead>
<tr>
<th>N</th>
<th>DCR</th>
<th>MF</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>38</td>
<td>40</td>
</tr>
<tr>
<td>25</td>
<td>405</td>
<td>188</td>
</tr>
<tr>
<td>35</td>
<td>2925</td>
<td>487</td>
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<tr>
<td>55</td>
<td>51128</td>
<td>1159</td>
</tr>
<tr>
<td>65</td>
<td>144566</td>
<td>3089</td>
</tr>
</tbody>
</table>

see if the slight overdimensioning of the MF network produced any significant differences in terms of network performance. The first result we present in Fig. 7 is the total network loss probability—that is, the total fraction of the offered traffic that is rejected from the network—for each state, both for the MF network and the DCR network. In the figure, the horizontal axis indicates the number of the failed transmission link whereas the vertical axis shows the total end-to-end loss probability (averaged over all O–D pairs) for each state considered (failed link number 0 denotes the state without failure). The physical links were numbered in lexicographic order. For each failure considered, the total weighted blocking probability of all O–D pairs were plotted for the MF and DCR networks. This result shows that the assumption made in some references given before that the adaptive nature of the DCR routing algorithm is sufficient to ensure reliability under failure is, in fact, incorrect. As we will see later, some care must be given to the reliability aspect in the dimensioning process even if we plan to use an adaptive routing technique.

From the figure, we see that the DCR network presented systematically higher values of loss probability than the MF network in the failure states. The most significant discrepancies between the two sets of results were found for the failure of physical links (2–12) and (2–18) (labeled, respectively, as link 8 and 9 in the figure). Detailed results on those failures are shown in Figs. 8 and 9, respectively.

Each figure shows the end-to-end loss probabilities versus the O–D’s for the MF and DCR networks. For ease of reading, we have again numbered every O–D in lexicographic order. In Fig. 8, the highest differences between the two networks performance results occur for O–D numbered 8–17. For those O–Ds, the loss probability in the DCR network was about twice as high as the corresponding value in the MF network. The differences were less concentrated, but equally striking, in Fig. 9. In this figure it can be seen that, when physical link (2–18) fails, the loss for some of the O–D’s (O–D number 10 is a clear example) is almost three times as high for the DCR network as for the MF network. The MFsyn algorithm has produced a network that allows the DCR routing to keep the global network blocking under control and, at the same time, ensures that some particularly sensible O–D’s maintain a reasonable level of loss under failure.

D. Effect of Failures on Direct Call Routing

To further investigate the performance of the two networks under failure, we have obtained some results concerning the distribution of traffic in the two networks when physical link (2–12) has failed. As before, the network performance was evaluated using the DCR call routing algorithm in both networks. Figs. 10 and 11 both depict the fraction of the offered traffic that is carried on the direct link, for every O–D, for the no-failure state and the state of (2–12) failed link. Fig. 10 shows the results for the DCR network, and Fig. 11 for the MF network. In both figures, it is clear that when no failures are present most of the traffic is routed through the direct link, as is well known.

In the event of failures, the routing for the DCR network changes dramatically. In Fig. 10, it is clear that for the two failures considered, the behavior of the DCR routing algorithm is strikingly different in the presence of failures. In particular, the failure of link (2–12) increases significantly the amount of alternate routing of most O–D pairs. This seems to imply that there is not enough capacity in the direct link in the event of failures. The same effect is present in the MF network, but to a smaller degree.

The behavior of the DCR routing algorithm on the two networks can be directly compared in Fig. 12. On this scatter plot, each point represents a particular O–D pair. The x coordinate indicates the fraction of direct-routed traffic in the
MF network and the y coordinate indicates the fraction in the DCR network. We see that the DCR algorithm routes more traffic on alternate routes when it operates on the DCR network than it does when operating on the MF network. This is due to the slight overdimensioning of the MF network that comes from the fact that reliability factors are incorporated into the synthesis procedure. In fact, it is rather surprising that such a small difference in trunk group sizes, as shown in Fig. 5, could have such a significant effect on traffic routing in the presence of failures. It simply reflects the fact that a slight overprovisioning, such as provided by the MF dimensioning algorithm, is sufficient to increase the amount of direct routing significantly, which is known to be generally beneficial to the grade of service. We can also see from Fig. 12 that the O–D pairs seem to cluster in two groups. We suspected that the difference was related to the routing of the trunk group in the transmission network. In order to check this, we have made a series of runs with the random networks. We have made the distinction between O–D pairs that were affected by a failure and those that were not. A typical result is presented in Fig. 13. The scatter plots for all of the networks that we tested (about 15) were basically identical to the one of Fig. 13.

E. Overflow and Failure Propagation Effect

Another insight on the effect of failures on routing performance is given by Figs. 14–16. First of all, we noted that despite the fact that the circuits of trunk group (2–9) are not routed over physical link (2–12), the loss probability of the traffic stream between 2 and 9 is highly influenced by the failure of link (2–12). We suspect that this is due to the overflow from other trunk groups that are routed through (2–12) and thus increase their overflow on trunk group (2–9) when the (2–12) link fails. In Figs. 14 and 15, the selection probability of alternate routes are shown for all flows originating at node 2 that use trunk group (2–9) as overflow [this is the value of the $J_{92}^j$ coefficient from (8)]. Fig. 14 presents the situation for the DCR network and Fig. 15 for the MF network. In each figure the selection probabilities are portrayed versus the node used as tandem with and without a failure of link (2–12).
For both cases, it is clear that the selection probabilities greatly increase in the case of failure and that it is this increase in alternate-routed traffic of other pairs that degrades the grade of service for the traffic stream (2–9). In Fig. 16, the selection probabilities for the MF network and the DCR network in the event of failure (2–12) are both shown. It seems clear from this figure that the increase in the selection probabilities is greater for the DCR network than for the MF network. This is another result suggesting that the effect of failure propagation is greater in the network that was not designed with failures in mind than in the network that was designed taking failures into account.

F. Effect of Circuit Routing

The results presented so far were all obtained with the same circuit routing rule: Each trunk group is routed in equal parts on the two arc-disjoint shortest paths in the transmission network. We have investigated the effect of this equal routing rule by changing the proportions to a 2:1 and a 3:1 ratio between the shortest and second shortest paths (see Table II for details on the multipliers). The results obtained were essentially the same as those obtained for an equal partition in the physical network is also the subject of optimization, along the lines of [26] and [27]. We must also point out one limitation due to our choice of a node–link representation for the MF formulation. In many networks, a maximum of two links is imposed on the paths used for alternate routing for various traffic engineering reasons. An arc-path formulation would then be more appropriate to model the limitations of path length since it is possible to enumerate all two-link paths for each O–D pair, and also because the subproblem generated by the Frank–Wolfe method is straightforward.

VII. CONCLUSION

We have provided a methodology to evaluate how MF models can be used for network synthesis with reliability. First of all, we have proposed a procedure to compare the performance of a real-time adaptive call routing algorithm and a multicommodity model. We have seen that the MF is substantially different from the actual flows produced by the real-time routing.

We have also shown that using an adaptive routing method alone is not sufficient to ensure a reliable performance in the presence of failures and that MF models can be used for network synthesis when taking into account failure propagation. They require relatively little computation and they significantly improve, at the price of a small cost increase, the robustness of the resulting networks. In fact, because of their efficiency, MF algorithms can dimension a network taking into account all significant failure states faster than a DCR-based method that takes no account of reliability, and this difference in computation times gets larger as the network size increases. The only difficulty with the approach is that it is currently difficult to tightly control the grade of service in all failed states.

This work has opened the door to further research concerning the impact of failures in adaptive routing. First of all, we could conceive of DCR dimensioning models with reliability constraints for all failure states explicitly incorporated. If an optimal solution is found, it will necessarily have the proper loss probabilities in all states. Preliminary work, however, indicates that the computation time of such methods could be extremely large. This, in turn, suggests that a hybrid model where a first approximation could be evaluated with the MF model and the DCR model used as a final refinement to get better control over the grade of service constraints if needed.

Concerning failure propagation, our results confirm that even when an adaptive routing is used in real time, there are three interrelated factors responsible for a degraded performance. First of all, there is the inevitable loss of capacity for the O–D’s that use the failed link. The second factor is a clear decrease of direct call routing for all O–D’s, whether they are directly or indirectly affected by the failure. Finally, we have clearly seen, as was also shown in [27], that the overflow from a given traffic stream directly affected by the failure can degrade the performance of other traffic streams, even those whose direct link is routed on paths that do not use the failed link. We have shown that multicommodity models produce a slight overdimensioning that, in turn, provides an excellent way to control the effect of the negative factors previously described.

In our modeling, we have worked from the physical failure up to the trunk dimensioning. That is, we supposed that the trunk routing is fixed and that only the call routing varies in the procedure. The next step should be to consider using MF’s in integrated models in which the trunk group routing in the physical network is also the subject of optimization, along the lines of [26] and [27]. We must also point out one limitation due to our choice of a node–link representation for the MF formulation. In many networks, a maximum of two links is imposed on the paths used for alternate routing for various traffic engineering reasons. An arc-path formulation would then be more appropriate to model the limitations of path length since it is possible to enumerate all two-link paths for each O–D pair, and also because the subproblem generated by the Frank–Wolfe method is straightforward.

Another limitation of the approach is the fact that state protection was not considered in the modeling. State protection can produce a reduction in blocking for networks in overloaded conditions that are operated with DCR. Therefore, the conclusions from our study cannot be automatically extended to networks with state protection.

Finally, we mention that the research presented here is being extended to the synthesis of multiservice broad-band networks.

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REFERENCES


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