Digital Modulation – Lecture 05

M-ary systems
Problems in the wireless environment: noise, interference and the mobile channel

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Objectives

• To be able to describe each of the following M-ary signalling schemes and to analyse their PSD and related properties:
  – QPSK, MPSK
  – QAM
  – OQPSK, $\pi/4$ QPSK
  – MSK, GMSK

• You will be able to:
  – Discuss the causes of noise in communication systems
  – Describe how performance is degraded in mobile / radio communication systems
Presentation Outline

• More detailed view of M-ary signalling schemes
  – QPSK, MPSK
  – QAM
  – OQPSK, π/4 QPSK
  – MSK, GMSK

• Introduction to the concepts of noise in communication systems (radio system emphasis).
Introduction to Multilevel Signalling

- Binary keying produces two distinct signals (symbols) that represent the 1s and 0s in the message.
- In this case, the symbol rate is equal to the bit rate of the data stream.
- If we could define a symbol that represented two (or more) bits at a time, we could improve the data throughput.
- For example, if each symbol stands for 2 bits, the throughput could be doubled.
- We need four symbols to encode, 00, 01, 10 and 11
If we wanted to triple the throughput, we need symbols for the combinations 000, 001, 010, 011, 100, 101, 110 and 111 a total of 8 symbols are required.

- Sounds like a good idea, but.....
- As the number of symbols increases, the penalty is more noise-related errors can cause problems.
- This general idea is referred to as M-ary Modulation
M-ary Signalling

• It is relatively easy to create four unique symbols:
  – Four amplitude values (Quadrature ASK)
  – Four phases values (Quadrature PSK)
  – Four frequency values (Quadrature FSK)
• To produce more complex symbols, that represent larger groups of bits, the amplitude, phase or frequency of the carrier can be used alone, or in combination with either a single carrier or a coherent quadrature-carrier arrangement.
• Known as M-ary signalling, each symbol represents $\log_2 M$ bits so that the bit rate is $r \log_2 M$.
• $r$ is the symbol rate (symbols/sec)
• $M$ has values like 2, 4, 8, 16 etc
Among the many examples of M-ary signalling schemes we have already seen the following:

- QPSK, MPSK
- OQPSK, $\pi/4$ QPSK
- MSK, GMSK
• **Quadrature Amplitude Modulation**
  – Unlike MPSK, QAM constellations are not restricted to a circle
    
    \[ s(t) = x(t)\cos \omega_c t - y(t)\sin \omega_c t \]

  – Where \( g(t) = x(t) + jy(t) \)

  – Example: 16-symbol QAM (16 levels)

• QAM bits are transmitted at rate 2r/s in the bandwidth required by Binary ASK.
• Note that $x$ and $y$ are permitted to have four levels
Generation of QAM Signals

(a) Modulator for Generalized Signal Constellation

(b) Modulator for Rectangular Signal Constellation

Figure 5-31 Generation of QAM signals
QPSK and MPSK – 1

• Quadrature Phase-Shift Keying and M-ary Phase Shift Keying
  – The modulated signal is an M-level digital signal
  – When M=4 we have called QPSK
  – The complex envelope is given by
    \[ g(t) = A_c e^{j\theta(t)} \]
  – Each level of the modulating signal corresponds to a distinct phase for the allowable \( \theta \).
QPSK and MPSK – 2

• Example: Two QPSK signal constellations

(a) $\theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ$
(b) $\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$.

• Note that the signalling points lie on a circle.
The permitted values of $\theta$ are $\theta_i = 1, 2, 3, \ldots, M$

- MPSK can be generated using a phase modulator
- MPSK can also be generated using two quadrature carriers

$$g(t) = A_c e^{j\theta(t)} = x(t) + jy(t)$$

- where $x(t)$ is called the I component and $y(t)$ is called the Q component

- The permitted values of $x$ and $y$ are:

$$x_i = A_c \cos \theta_i, \ y_i = A_c \sin \theta_i$$

$$s(t) = x(t) \cos \omega_c t - y(t) \sin \omega_c t$$
Generation of MPSK - 2
Offset QPSK

- Offset OPSK is defined as follows:
  - The I and Q components are offset by $\frac{1}{2}$ symbol (i.e., 1 bit) interval
  - Since I and Q cannot change simultaneously, the maximum phase transition is 90° as opposed to 180° for QPSK.
  - It reduces the amplitude fluctuation (which is proportional to the degree of phase transition) due to filtering in the course of transmission.
\( \pi/4 \) QPSK

- This is generated by alternating between two QPSK constellations that are rotated by \( \pi/4 \) radians = 45° with respect to each other.
- Possible phase transitions are ±45° and ±135° as opposed to ±90° and ±180° for QPSK.
- It is usually differentially encoded, ie \( \pi/4 \) DQPSK

<table>
<thead>
<tr>
<th>Data</th>
<th>Phase Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>-135°</td>
</tr>
<tr>
<td>01</td>
<td>+135°</td>
</tr>
<tr>
<td>10</td>
<td>-45°</td>
</tr>
<tr>
<td>11</td>
<td>+45°</td>
</tr>
</tbody>
</table>
The complex envelope of MPSK and QAM can be written as

\[ g(t) = \sum_{n=-\infty}^{\infty} c_n f(t - nT_s) \]

- Where \( c_n \) is a complex value representing the nth symbol according to the constellation, \( T_s \) is the symbol duration, \( f(t) \) is the symbol pulse shape.

For rectangular symbol pulse shapes, \( f(t) = \Pi(t/T_s) \), it is found that the PSD of \( g(t) \) is given by:

\[ P_g(f) = K \left( \frac{\sin \pi f \ell T_b}{\pi f \ell T} \right)^2 \]

- Where \( K = 2P\ell T_b \) and \( P \) is the total transmitted power.
The PSD of MPSK and QAM - 2

- PSD for the complex envelope of MPSK and QAM with rectangular data pulses, where $M = 2^\ell$, $R$ is the bit rate and $R/\ell$ is the baud rate. (Positive frequencies shown)
- Use $\ell=2$ for PSD of QPSK, OQPSK and p/4 QPSK complex envelopes
Recalling our previous definition of spectral efficiency, we can now determine this for MPSK and QAM based on rectangular pulses. For rectangular pulses, the null-to-null transmission bandwidth of MPSK or QAM is

\[ B_T = \frac{2R}{\ell} \]

Therefore the spectral efficiency is

\[ \eta = \frac{R}{B_T} = \frac{\ell \text{ bits/sec}}{2 \text{ Hz}} \]
Spectral Efficiency – 2

- With raised cosine filtering the absolute bandwidth of the multilevel modulating signal is
  \[ B_T = \frac{1}{2} (1 + r)D \]

- Thus the absolute transmission bandwidth is
  \[ B_T = 2B = (1 + r) \frac{R}{\ell} \]

- And the spectral efficiency is
  \[ \eta = \frac{R}{B_T} = \frac{\ell}{1 + r} = \frac{\log_2 M}{1 + r} \left( \frac{\text{bits/s}}{\text{Hz}} \right) \]
Spectral Efficiency – 3

• According to Shannon’s theorem, \( R < C \), ie

\[
\eta B_T < B_T \log_2 \left( 1 + \frac{S}{N} \right)
\]

• Hence \( \eta < \eta_{\text{max}} \)

• Where

\[
\eta_{\text{max}} = \log_2 \left( 1 + \frac{S}{N} \right)
\]

• For a given power, as \( M \) is increased, the spacing between the signal points in the constellation will increase and the noise on the received signal will cause errors.
Comparison of Modulation Schemes

- This graph shows that bandwidth efficiency is traded off against power efficiency.
- MFSK is power efficient, but not bandwidth efficient.
- MPSK and QAM are bandwidth efficient but not power efficient.
- Mobile radio systems are bandwidth limited, therefore PSK is more suited.
## Comparison of Modulation Types

<table>
<thead>
<tr>
<th>Modulation Format</th>
<th>Bandwidth efficiency (C/B)</th>
<th>$\log_2$ (C/B)</th>
<th>Error free Eb/No</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 PSK</td>
<td>4</td>
<td>2</td>
<td>18dB</td>
</tr>
<tr>
<td>16 QAM</td>
<td>4</td>
<td>2</td>
<td>15dB</td>
</tr>
<tr>
<td>8PSK</td>
<td>3</td>
<td>1.585</td>
<td>14.5dB</td>
</tr>
<tr>
<td>4PSK</td>
<td>2</td>
<td>1</td>
<td>10.1dB</td>
</tr>
<tr>
<td>4QAM</td>
<td>2</td>
<td>1</td>
<td>10.1dB</td>
</tr>
<tr>
<td>BFSK</td>
<td>1</td>
<td>0</td>
<td>13dB</td>
</tr>
<tr>
<td>BPSK</td>
<td>1</td>
<td>0</td>
<td>10.5dB</td>
</tr>
</tbody>
</table>
Introduction to Noise

Problems in the wireless environment: noise, interference and the mobile channel
Noise in the Mobile Radio Channel

- Noise arises from a variety of sources, including automobile ignitions and lightning, or thermal noise in the receiver itself. Thermal noise can be modelled as Additive White Gaussian Noise (AWGN).
- The ratio of the signal strength to the noise level is called the signal-to-noise ratio (SNR). If the SNR is high (i.e., the signal power is much greater than the noise power) few errors will occur. However, as the SNR reduces, the noise may cause symbols to be demodulated incorrectly, and errors will occur.
- The bit error rate (BER) of a system indicates the quality of the link.
- Usually, a BER of $10^{-3}$ is considered acceptable for a voice link, and $10^{-9}$ for a data link. A coherent QPSK system requires an SNR of greater than approximately 12dB for a BER of better than $10^{-3}$. 
Interference in the Mobile Radio Channel

- **Interference** is the result of other man-made radio transmissions.
  - for example in the ISM band at 2.4GHz a large number of systems co-exist, such as Wireless LAN, Bluetooth, Microwave ovens, etc

- **Adjacent channel interference** occurs when energy from a carrier spills over into adjacent channels.

- **Co-channel interference** occurs when another transmission on the same carrier frequency affects the receiver. This will often arise from transmissions in another cell in their network.

- The ratio of the carrier to the interference (from both sources) is called the *carrier-to-interference ratio* (C/I). A certain C/I ratio is required to provide adequate quality transmission.
  - Increasing the carrier power at the receiver will increase the interference for other mobiles in the network.
The Multipath Environment

- The received signal is made up of a sum of attenuated, phase shifted and time delayed versions of the transmitted signal.
- Propagation modes include diffraction, transmission and reflection.
Narrowband Fast Fading

- If time dispersion is small, vector sum of rays occurs.
- The arrival phase of each path alters as the receiver moves, resulting in a different vector sum.
- Rapid phase and amplitude shifts are observed (up to 40dB).
- Magnitude is modelled as Rayleigh (no line-of-sight) or Rician (more deterministic).
Shadowing (Slow Fading)

- Amplitude variation occurs as the receiver moves behind buildings and the propagation paths are obscured.
- Variations of up to 20dB will cause handovers and change the quality-of-service.
The received signal in a multipath channel exhibits large variations in magnitude.

Although the mean SNR (or C/I) might be acceptable, the variations experienced in the multipath channel mean that occasionally the noise will be far more significant. At these times the system will experience a large number of errors.
Rayleigh Distribution of the Multipath Channel

- A multipath channel without a significant deterministic component can be approximated to a Rayleigh distribution.
- The received signal experiences large variations in magnitude. For example, there is a 0.1% chance of the signal being 30dB below the mean level.
- Consequently, even a system with a high SNR can experience errors as the signal fades.
System Performance in AWGN

- The effects of the multipath channel (Rayleigh fading) severely degrade the system performance in the presence of Additive White Gaussian Noise.